Dynamical Symmetry Breaking by SU(2) Gauge Bosons

F.J. Himpsel

Department of Physics, University of Wisconsin Madison,
1150 University Ave., Madison, WI 53706, USA, fhimpsel@wisc.edu

Abstract

This work explores the possibility of creating mass in Yang-Mills gauge theories via their intrinsic gauge bosons, rather than by an additional Higgs boson. Instead, pairs of gauge bosons in the spin and isospin singlet state form a pair of composite Higgs bosons. Those pairs can be viewed as glueballs in Yang-Mills gauge theories, with the SU(2) gauge group as the simplest example. Quadratic and quartic gauge boson self-interactions form a potential that leads to a finite expectation value for each virtual gauge boson in the vacuum state. The Lorentz invariance of the vacuum is protected after averaging over all possible polarization vectors (analogous to averaging over all momenta). But the scalar pair products of gauge bosons used in the definition of the composite Higgs boson exhibit a finite vacuum expectation value. That breaks the gauge symmetry dynamically and thereby creates masses for the gauge bosons. In the standard model, the ad-hoc potential of the Higgs boson can be replaced by the intrinsic quadratic and quartic self-interactions of the gauge bosons, thereby eliminating two free parameters. These can be taken as the mass and the vacuum expectation value of the standard Higgs boson.

1. Introduction

Calculating adjustable parameters of the standard model from first principles has been a long-time challenge. Particularly mysterious have been the two parameters determining the Higgs potential of the standard model. One of them (labeled $-\mu^2$) corresponds to an imaginary mass and the other (labeled λ) belongs to a quartic Lagrangian. None of the other fundamental particles exhibits such a Lagrangian. This *ad-hoc* potential is responsible for breaking the $SU(2)\times U(1)_Y$ gauge symmetry of the electroweak interaction by creating a finite vacuum expectation value (VEV) for the Higgs boson. That in turn conveys mass to fundamental particles.

Such considerations led to models where the Higgs boson is not fundamental, but composite [1],[2]. In most cases the constituents were fermion-antifermion pairs [1], but a Higgs boson composed of the three SU(2) gauge bosons was proposed as well [2]. It explained the Higgs mass, which became simply half of the standard Higgs VEV in lowest order. That matched the experimental result [3] with tree-level accuracy ($\approx 2\%$).

Apart from exploring the origin of mass in particle physics, the SU(2) gauge theory has attracted interest in mathematical physics [4]. The three SU(2) gauge bosons form the simplest non-abelian gauge theory. Such Yang-Mills theories play a dominant role in the standard model and its extensions. A particular concern has been the very existence of such theories (by rigorous mathematical standards), together with the mechanism of dynamical symmetry breaking and the resulting mass gap [4].

In the following we start out with a review of the composite Higgs model proposed in [2], restricted to the minimal set of particles, i.e., the three gauge bosons of the SU(2) Yang-Mills theory. The definition of the composite Higgs boson via these gauge bosons is worked out and its consequence on the Higgs mass is demonstrated. This section establishes several relations between the composite Higgs boson and the SU(2) gauge bosons. Section 3 investigates the precise form of the expectation value (EV) for the amplitude of a gauge field, as dictated by Lorentz and gauge invariance. The gauge fields in the Lagrangian are decomposed into their EVs and observable gauge bosons. Section 4 introduces the gauge boson potential which breaks the SU(2) gauge symmetry and creates VEVs for gauge boson pairs. Gauge-invariant model Lagrangians serve as building blocks for this potential. Section 5 exploits compatibility criteria between the potentials of the gauge bosons and the Higgs boson. Those fix the quadratic and quartic coupling constants. Section 6 summarizes the concept of dynamical symmetry breaking by gauge bosons and places it into the broader context of Yang-Mills theories. Appendix A discusses tree-level gauge boson self-interactions and explains why they do not contribute to the symmetry-breaking potential. Appendix B outlines the extension of the SU(2) model to the electroweak $SU(2)\times U(1)_Y$ symmetry.

2. Composite Higgs Model for Pure SU(2) Gauge Symmetry

The pure SU(2) model is chosen to provide clearer insight into the concept of a Higgs boson composed of gauge bosons which was developed originally for the full $SU(2)\times U(1)_Y$ electroweak symmetry in [2]. Mixing with the $U(1)_Y$ hypercharge symmetry does not affect the W^1,W^2 gauge bosons that form the observed $W^{\pm}$ particle, leaving their mass and couplings unchanged. The consequences of electroweak mixing will be addressed briefly in Appendix B.

The strategy for replacing the Higgs boson of the standard model by a composite of SU(2) gauge bosons can be summarized as follows:

1) Remove the standard Higgs boson from the Lagrangian.
2) Replace it by a Lorentz- and gauge-invariant composite of SU(2) gauge bosons.
3) Establish a potential for the gauge bosons from their self-interactions.
4) Obtain EVs for the gauge bosons and VEVs for their scalar products by generalizing the Brout-Englert-Higgs mechanism from scalars to vectors.
5) Transfer VEVs and masses from the gauge bosons to the composite Higgs boson.

The standard Higgs field can be written as the combination of a SU(2) singlet H_0 with a triplet of Nambu-Goldstone modes (w_1,w_2,w_3), forming a complex doublet Φ_0. The subscript 0 labels fields appearing in the original, gauge-invariant Lagrangian ("lagrangian fields"). These are decomposed into a VEV and an observable field. The Higgs field Φ_0 can be represented by a 2×2 matrix $\boldsymbol{\Phi}_0$ using the 2×2 unit matrix $\mathbf{1}$ and the Pauli matrices $\boldsymbol{\tau}^j$ (with 2×2 matrices in bold):

$$(1) \qquad \boldsymbol{\Phi}_0 = \tfrac{1}{\sqrt{2}}(H_0\cdot\mathbf{1} + i\,\Sigma_j\, w_j\cdot\boldsymbol{\tau}^j) \qquad \Phi_0 = \boldsymbol{\Phi}_0\cdot\begin{bmatrix}0\\1\end{bmatrix} = \tfrac{1}{\sqrt{2}}\begin{bmatrix}w_2+i\,w_1\\ H_0-i\,w_3\end{bmatrix} \qquad \Phi_0^C = \boldsymbol{\Phi}_0\cdot\begin{bmatrix}1\\0\end{bmatrix}$$

$$\mathrm{H}_0 = \langle \mathrm{H}_0 \rangle + \mathrm{H} \qquad \langle \mathrm{H}_0 \rangle = v = 2^{-1/4}\, G_\mathrm{F}^{-1/2} = 246.22\,\mathrm{GeV} \qquad \langle w_\mathrm{j} \rangle = 0$$

Φ_0^C is the charge conjugate of Φ_0. The singlet acquires a finite VEV $\langle \mathrm{H}_0 \rangle = v$ via the Brout-Englert-Higgs mechanism. Its value is directly related to the experimental four-fermion coupling constant G_F [3]. The VEVs of the Goldstone modes vanish.

The standard Higgs potential combines a quadratic with a biquadratic term:

$$(2) \qquad \mathrm{V}_\Phi = -\mu^2 \cdot \Phi_0^\dagger \Phi_0 + \lambda \cdot [\Phi_0^\dagger \Phi_0]^2 \qquad \text{General gauge}$$

$$\mathrm{V}_\Phi = -\tfrac{1}{2}\mu^2 \cdot \mathrm{H}_0^2 + \tfrac{1}{4}\lambda \cdot \mathrm{H}_0^4 \qquad \text{Unitary gauge}$$

Using the pairs $\Phi_0^\dagger \Phi_0$ or H_0^2 as variables simplifies the potential to a linear plus a quadratic term, providing a hint that pairs may play a role in Higgs interactions.

The SU(2) gauge bosons form a triplet $(\mathrm{W}_0^1, \mathrm{W}_0^2, \mathrm{W}_0^3)$. The sum over gauge boson pairs $\Sigma_\mathrm{i} \mathrm{W}_{0,\mu}^\mathrm{i} \mathrm{W}_0^{\mathrm{i},\mu}$ is a Lorentz scalar and a SU(2) singlet, thereby matching $\Phi_0^\dagger \Phi_0$. That suggests a proportionality between a pair of Higgs bosons and pairs of gauge bosons:

$$(3) \qquad \Phi_0^\dagger \Phi_0 = \tfrac{1}{2}[\mathrm{H}_0^2 + \Sigma_\mathrm{i} w_\mathrm{i}^2] \propto -\tfrac{1}{2}\Sigma_\mathrm{i} \mathrm{W}_{0,\mu}^\mathrm{i} \mathrm{W}_0^{\mathrm{i},\mu} = -\tfrac{1}{2}\Sigma_\mathrm{i} (\mathrm{W}_0^\mathrm{i} \mathrm{W}_0^\mathrm{i}) \qquad (+---)\ \text{metric}$$

Using the equal sign gives each bosonic degree of freedom the same weight. The minus sign compensates for the negative scalar products of the space-like gauge bosons.

Instead of defining $\mathbf{\Phi}_0$ via (1), the Goldstones w_i can be incorporated in nonlinear fashion as SU(2) matrix $\mathbf{U}$ (see [5], including a note regarding $\mathbf{V}_{0,\mu}$ defined below):

$$(4) \qquad \mathbf{U} = \exp\left(i \cdot \Sigma_\mathrm{j} \tfrac{w_\mathrm{j}}{v} \cdot \boldsymbol{\tau}^\mathrm{j}\right) = \cos(\mathrm{x}) \cdot \mathbf{1} + i \cdot \sin(\mathrm{x}) \cdot \mathbf{y}/\mathrm{x} \qquad \mathrm{x} = \tfrac{|w|}{v} \qquad |w| = (\Sigma_\mathrm{i} w_\mathrm{i}^2)^{1/2}$$

$$= \mathbf{1} + i\,\mathbf{y} - \tfrac{1}{2}\mathrm{x}^2 \cdot \mathbf{1} - \tfrac{1}{6}\mathrm{x}^2 \cdot i\,\mathbf{y} + \mathrm{O}(\mathrm{x}^4) \qquad \mathbf{y} = \Sigma_\mathrm{j} \tfrac{w_\mathrm{j}}{v} \cdot \boldsymbol{\tau}^\mathrm{j}$$

$$\tfrac{1}{\sqrt{2}} \mathrm{H}_0 \cdot \mathbf{U} \approx \mathbf{\Phi}_0 \quad \text{for} \quad \tfrac{|\mathrm{H}|}{v}, \mathrm{x} \ll 1$$

The gauge bosons are incorporated via the gauge-invariant derivative D_μ of the matrix $\mathbf{U}$:

$$(5) \qquad i\mathrm{D}_\mu \mathbf{U} = i\partial_\mu \mathbf{U} + g(\Sigma_\mathrm{j} \mathrm{W}_{0,\mu}^\mathrm{j} \cdot \tfrac{1}{2}\boldsymbol{\tau}^\mathrm{j}) \cdot \mathbf{U} \qquad \partial_\mu \mathrm{x} = \Sigma_\mathrm{j} \tfrac{w_\mathrm{j}}{|w|} \partial_\mu \tfrac{w_\mathrm{j}}{v}$$

A four-vector of hermitian 2×2 matrices $\mathbf{V}_{0,\mu}$ represents the gauge bosons $\mathrm{W}_{0,\mu}^\mathrm{j}$:

$$(6) \qquad \mathbf{V}_{0,\mu} = (i\mathrm{D}_\mu \mathbf{U})\mathbf{U}^\dagger = \tfrac{1}{2} g \Sigma_\mathrm{j} \mathrm{W}_{0,\mu}^\mathrm{j} \boldsymbol{\tau}^\mathrm{j} - \partial_\mu \mathbf{y} - i\mathrm{x}\partial_\mu \mathrm{x} \cdot \mathbf{1} + (\tfrac{1}{3}\mathrm{x}\partial_\mu \mathrm{x} \cdot \mathbf{y} + \tfrac{1}{6}\mathrm{x}^2 \cdot \partial_\mu \mathbf{y}) + \mathrm{O}(\mathrm{x}^4)$$

$$(7) \qquad \mathrm{tr}[(\mathbf{V}_0 \mathbf{V}_0)] = \tfrac{1}{2} g^2 \cdot \Sigma_\mathrm{i} \mathrm{W}_{0,\mu}^\mathrm{i} \mathrm{W}_0^{\mathrm{i},\mu} + 2\Sigma_\mathrm{i} \partial_\mu \tfrac{w_\mathrm{i}}{v} \partial^\mu \tfrac{w_\mathrm{i}}{v} + \mathrm{O}(\mathrm{x}^4)$$

Notice that the Goldstone terms contain the longitudinal vectors $i\partial_\mu \frac{w_\mathrm{i}}{v} \to \mathrm{k}_\mu \frac{w_\mathrm{i}}{v}$, while the gauge bosons $\mathrm{W}_{0,\mu}^\mathrm{j}$ are purely transverse. As a result, their mixed products vanish in (7). Also, the mixed products between the matrices $\mathbf{y}$ and $\mathbf{1}$ have vanishing trace.

Equation (7) provides a gauge-invariant definition of the composite Higgs boson:

(8) $\boxed{\Phi_0^\dagger\Phi_0 = -\mathrm{tr}[(\mathbf{V}_0\mathbf{V}_0)]}$ General gauge

(9) $\tfrac{1}{2}\mathrm{H}_0^2 + \tfrac{1}{2}\Sigma_i w_i^2 = -g^2\cdot\tfrac{1}{2}\Sigma_i(\mathrm{W}_0^i\mathrm{W}_0^i) - 2\Sigma_i\,\partial_\mu\frac{w_i}{v}\cdot\partial^\mu\frac{w_i}{v} + \mathrm{O}(\mathrm{x}^4)$

(10) $\boxed{\tfrac{1}{2}\mathrm{H}_0^2 = -g^2\cdot\tfrac{1}{2}\Sigma_i(\mathrm{W}_0^i\mathrm{W}_0^i)}$ Unitary gauge

Notice a subtle, but conceptually-significant difference from Ref. [2], where the composite Higgs boson was defined in terms of observable gauge bosons. Here we start with lagrangian gauge bosons and discuss observable gauge bosons in Section 3. Making the definition in the Lagrangian preserves explicit SU(2) gauge symmetry.

The leading term on the left side of (9),(10) comes from the VEV of the Higgs boson $\langle\mathrm{H}_0\rangle^2 = v^2$, because the observable Higgs field H represents small oscillations about the VEV. Otherwise the VEV would not be noticed on top of the oscillations. This implies that the leading term on the right side must be due to finite VEVs of gauge boson pairs. A vector field with a finite VEV $\langle\mathrm{W}_0^i\rangle$ would violate Lorentz invariance of the vacuum by specifying a specific direction in space-time. This problem is avoided by having different orientations for the expectation values $\langle\mathrm{W}_0^i\rangle$ of individual field quanta, depending on their momenta [2]. These EVs change from one gauge boson to another and average out to zero after summing over all virtual gauge bosons in the vacuum of quantum field theory (compare the summation over the vacuum photons in the calculation of the Casimir effect). But the scalar products $(\langle\mathrm{W}_0^i\rangle\langle\mathrm{W}_0^i\rangle)$ do not average out. They provide a finite VEV to match that of the Higgs boson. The leading term of (9),(10) then establishes a relation between v^2 and scalar products of gauge boson EVs:

(11) $v^2 = -g^2\cdot\Sigma_i(\langle\mathrm{W}_0^i\rangle\langle\mathrm{W}_0^i\rangle)$ General gauge

$\boxed{v = \sqrt{3}\,g w}$ defining $\boxed{w^2 = -(\langle\mathrm{W}_0^i\rangle\langle\mathrm{W}_0^i\rangle)}$ for i = 1,2,3

The smaller terms of (9),(10) will be worked out at the end of Section 3, after switching from lagrangian bosons to observable bosons in (17). That will lead to an approximate (tree-level) relation between the mass terms of the observable Higgs and gauge bosons:

(12) $\boxed{\Phi^\dagger\Phi \approx -\mathrm{tr}[(\mathbf{V}\mathbf{V})]}$ General gauge

(13) $\tfrac{1}{2}\mathrm{H}^2 + \tfrac{1}{2}\Sigma_i w_i^2 \approx -g^2\cdot\tfrac{1}{2}\Sigma_i(\mathrm{W}^i\mathrm{W}^i)$

(14) $\boxed{\tfrac{1}{2}\mathrm{H}^2 \approx -g^2\cdot\tfrac{1}{2}\Sigma_i(\mathrm{W}^i\mathrm{W}^i)}$ Unitary gauge

An important consequence of (14) is the determination of the Higgs mass from the four-fermion coupling constant G_F in [2]. This can be seen after generating the mass Lagrangians by multiplying (14) with $-(\tfrac{1}{2}v)^2$:

$$(15)\quad L_M^H = -\tfrac{1}{2}M_H^2\cdot H^2 \quad \text{with} \quad \boxed{M_H \approx \tfrac{1}{2}v}$$

$$(16)\quad L_M^W = \tfrac{1}{2}M_W^2\cdot\Sigma_i(W^iW^i) \quad \text{with} \quad \boxed{M_W \approx \tfrac{1}{2}vg} \quad \Rightarrow\ L_M^H \approx L_M^W$$

With the tree-level gauge boson mass $M_W \approx \tfrac{1}{2}vg$ taken from the standard model, the tree-level Higgs mass becomes $M_H \approx \tfrac{1}{2}v = 2^{-5/4}G_F^{-1/2} = 123.1\,\text{GeV}$. That matches the experimental result of $125.1\,\text{GeV}$ within 2% [3]. A similar match exists between $M_W \approx \tfrac{1}{2}vg = 77.5\,\text{GeV}$ and the observed value of $80.4\,\text{GeV}$. Such a margin is typical for the tree-level approximation which neglects corrections of the order $\alpha_w = g^2/4\pi \approx 3\%$.

3. Expectation Values of Gauge Bosons

The derivation of (12)-(14) from (8)-(10) calls for a more detailed analysis of the gauge boson EVs. They have to be transverse for two related reasons: 1) Only the transverse modes are gauge-invariant, while the longitudinal mode is traded for a Goldstone scalar when going from the unitary gauge to the Landau gauge. 2) A gauge-symmetric Lagrangian requires massless gauge bosons, which are purely transverse. These arguments also apply to their EVs. One can then decompose the lagrangian gauge bosons W_0^i into their EVs $\langle W_0^i\rangle$ and the transverse component of the observable gauge bosons: $W_0^i = \langle W_0^i\rangle + W_T^i$.

To obtain the polarizations of the gauge bosons contained in the composite Higgs boson we start at its definition, which involves pairs of identical gauge bosons. In a pair they move around their center of mass with opposite momenta. Those serve as reference for the polarizations/helicities. Both have the same circular polarization/helicity in this state. The wave function of the singlet ground state with even parity has the form $(\uparrow\uparrow+\downarrow\downarrow)/\sqrt{2}$. (For a more detailed description see [6],[14].) The three gauge boson pairs defining the Higgs boson all have the same polarization vector ε_α. They become products of ε_α with scalar operators w^i plus a common expectation value w. Similar to the Higgs boson, a SU(2) gauge boson is decomposed into an EV and an observable gauge boson:

$$(17)\quad W_0^i = \langle W_0^i\rangle + W^i \qquad \langle W^i\rangle = 0 \qquad \langle W_0^i\rangle = w\cdot\varepsilon_\alpha \qquad W_T^i = w^i\cdot\varepsilon_\alpha \qquad \text{General gauge}$$

$$(18) \qquad (\varepsilon^*_\alpha \varepsilon_\beta) = -\delta_{\alpha\beta} \qquad\qquad (W^i_T W^i_T) = -(w^i)^2$$

Only transverse components W^i_T appear explicitly. The longitudinal componens W^i_L are gauge-dependent and thus require gauge fixing. This is an elaborate procedure, but it has been performed routinely for the standard model and its extensions.

To obtain the relations (12)-(14) between the observable Higgs and gauge bosons we insert (17) into (9):

$$(19) \qquad \tfrac{1}{2} H_0^2 + \tfrac{1}{2}\Sigma_i w_i^2 = -g^2 \cdot \tfrac{1}{2}\Sigma_i (W^i_0 W^i_0) - 2\Sigma_i \partial_\mu \tfrac{w_i}{v} \cdot \partial^\mu \tfrac{w_i}{v} + \ldots \qquad \text{General gauge}$$

$$H_0^2 = v^2 + 2vH + H^2 \qquad (W^i_0 W^i_0) = (\langle W^i_0\rangle\langle W^i_0\rangle) + 2\cdot(\langle W^i_0\rangle W^i_T) + (W^i_T W^i_T)$$

It is tempting to identify the derivatives of the Goldstones w_i in the last term with longitudinal gauge bosons. Those vanish in the Landau gauge, where their role is played by the Goldstones.

There is a more general argument to justify the relations (12)-(14). These are between between the mass Lagrangians of the Higgs and gauge bosons. Since mass is an observable, they must be independent of the chosen gauge. However, they represent only the (dominant) tree-level masses and omit self-energy corrections. Therefore it is not surprising to see a deviation of a few % between the results in (15),(16) and the observed masses.

With the conversion from lagrangian to observable bosons in hand, one can expand the relation (10) between Higgs and gauge bosons into powers of their dominant quantities, i.e., their VEVs. The first three terms of the expansion have the form VEV·VEV, VEV·Boson, and Boson·Boson. The leading term produces the relation (11) between VEVs. The next-to-leading term provides a linear relation between observable Higgs and gauge bosons:

$$(20) \qquad vH \approx -g^2 \cdot \Sigma_i (\langle W^i_0\rangle W^i) \qquad \text{General gauge}$$

$$\boxed{H \approx (g/\sqrt{3}) \cdot \Sigma_i w^i \approx 0.38 \cdot \Sigma_i w^i} \qquad \text{via (11),(17)}$$

That makes it possible to replace Feynman diagrams containing the Higgs boson by diagrams for its constituents, the gauge bosons W^i. One can then proceed by analogy to other composite particles, such as replacing the proton by quarks and gluons. The third terms in the expansion leads to the relation (14) between pairs of observable bosons which determines the mass of the Higgs boson.

4. Symmetry-Breaking Gauge Boson Potential

In order to develop a finite VEV one needs a gauge-invariant potential that has a symmetry-breaking ground state. In the standard model, this is accomplished by the *ad-hoc* potential for the scalar Higgs boson which combines an attractive quadratic term with a repulsive quartic term. These are associated with two adjustable parameters $-\mu^2$ and λ. The SU(2) gauge bosons, on the other hand, exhibit non-abelian self-interactions which generate a suitable potential dynamically. These do not involve adjustable parameters (apart from the gauge coupling g which is also adjustable in the standard model). The gauge boson potential corresponds to the one-loop self-interactions shown in Figure 1. They contain quadratic and quartic terms analogous to the Higgs boson potential. For the diagrams in Fig. 1 these are of $O(g^2)$ of $O(g^4)$, respectively. They come with the effective coupling constants α_0 and α_5. Those can in principle be obtained by evaluating the diagrams in Figure 1, but such a calculation would go beyond the scope of this work. Instead, we will use compatibility with the standard Higgs potential to constrain them.

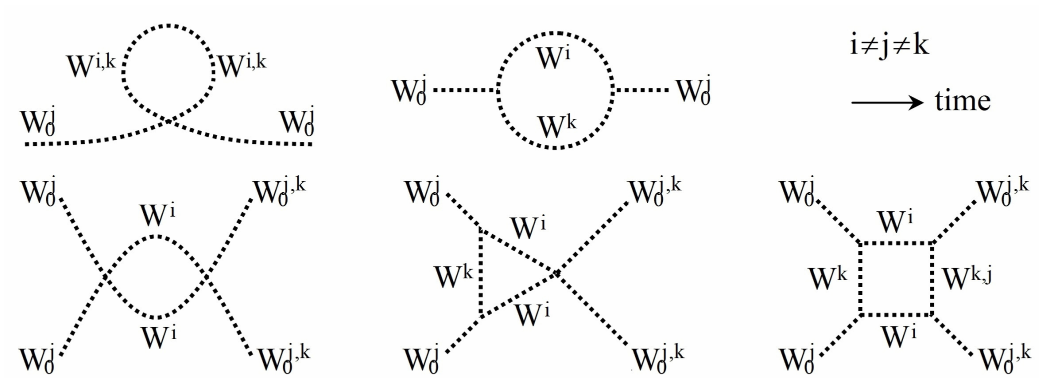

Figure 1 One-loop self-interactions of the SU(2) gauge bosons. These determine the symmetry-breaking potential. Top row: The quadratic self-energies of $O(g^2)$ which determine L_0,α_0. Bottom row: The quartic + biquadratic self-interactions of $O(g^4)$ which determine L_4,α_4 and L_5,α_5. They describe elastic scattering of gauge bosons [7],[8]. External lines correspond to lagrangian gauge bosons W_0^i which contain an expectation value (EV), while internal lines W^i lack an EV (see also Fig. 2 in Appendix A). To preserve the equivalence of the three gauge bosons W_0^i they have not been rearranged into $W_0^{\pm}$ and W_0^3.

One can make a generic ansatz for the gauge boson potential (and the corresponding Lagrangian) which satisfies gauge symmetry together with a custodial symmetry [5],[9]. The four-vector $\mathbf{V}_{0,\mu}$ defined in (4)-(6) generates three Lagrangians:

(21) $\boxed{L_0 = \alpha_0 \cdot \tfrac{1}{4} v^2 \cdot \mathrm{tr}[\mathbf{V}_{0,\mu}\mathbf{V}_0^{\mu}] \quad \rightarrow \quad \alpha_0 \cdot \tfrac{1}{4} g^2 v^2 \cdot \tfrac{1}{2} \Sigma_i (W_0^i W_0^i)}$ $\quad \tfrac{1}{4} v^2 \approx M_H^2 \quad \tfrac{1}{4} g^2 v^2 \approx M_W^2$

(22) $$\begin{array}{|l|} \hline L_4 = \alpha_4 \cdot \mathrm{tr}[\mathbf{V}_{0,\mu}\mathbf{V}_{0,\nu}] \cdot \mathrm{tr}[\mathbf{V}_0^{\mu}\mathbf{V}_0^{\nu}] \ \rightarrow \ \alpha_4 \cdot \tfrac{1}{4} g^4 \cdot \Sigma_{jk} (W_0^j W_0^k) \cdot (W_0^j W_0^k) \\ L_5 = \alpha_5 \cdot \mathrm{tr}[\mathbf{V}_{0,\mu}\mathbf{V}_0^{\mu}] \cdot \mathrm{tr}[\mathbf{V}_{0,\nu}\mathbf{V}_0^{\nu}] \ \rightarrow \ \alpha_5 \cdot \tfrac{1}{4} g^4 \cdot \Sigma_{jk} (W_0^j W_0^j) \cdot (W_0^k W_0^k) \\ \hline \end{array}$$

On the right side the Lagrangians have been reduced to the unitary gauge. To keep α_0 dimensionless, L_0 includes the scale factors M_H^2 and M_W^2 representing the squared tree-level masses of the Higgs and gauge bosons [9]. The factors g^2 and g^4 correspond to the diagrams in Fig. 1 top and bottom. While L_4 and L_5 consist of quartic+biquadratic terms, their difference L_{45} is purely biquadratic:

(23) $\boxed{L_{45} = \alpha_{45} \cdot (L_5/\alpha_5 - L_4/\alpha_4) \ \rightarrow \ \alpha_{45} \cdot \tfrac{1}{4} g^4 \cdot \Sigma_{j \neq k} [(W_0^j W_0^j) \cdot (W_0^k W_0^k) - (W_0^j W_0^k)^2]}$

L_{45} is proportional to the non-abelian gauge Lagrangian L_{bq} which describes the non-abelian vertex between four SU(2) gauge bosons (see Appendix A). It does not contribute to the potential of the composite Higgs boson. That leaves two independent Lagrangians for the dynamical gauge boson potential. These are chosen to be L_0, L_5:

(24) $\boxed{V^{dyn} = -(L_0 + L_5) = -\alpha_0 \cdot \tfrac{1}{4} v^2 \cdot \mathrm{tr}[(\mathbf{V}_0\mathbf{V}_0)] - \alpha_5 \cdot \{\mathrm{tr}[(\mathbf{V}_0\mathbf{V}_0)]\}^2}$

(25) $\boxed{V^{dyn} = -\alpha_0 \cdot \tfrac{1}{8} g^2 v^2 \cdot \Sigma_i (W_0^i W_0^i) - \alpha_5 \cdot \tfrac{1}{4} g^4 \cdot \{\Sigma_i (W_0^i W_0^i)\}^2}$ Unitary gauge

Choosing L_5 avoids mixed scalar products of the form $(W_0^j W_0^k)$ which would complicate the minimization of the potential. $-L_0$ is an attractive potential arising from the gauge boson self-energies (see [2]). It drives the potential minimum toward a finite EV. The quartic potential $-L_5$ must be repulsive to prevent a runaway of the potential minimum to $-\infty$ at large field amplitudes. To find the appropriate signs for the coupling constants one needs to take into account two minus signs, one from $V = -L$, and the other from the negative scalar products of space-like gauge bosons in $(\mathbf{V}_0\mathbf{V}_0)$ and $(W_0^i W_0^i)$. A potential with an attractive quadratic term and a repulsive quartic term requires $\alpha_0 < 0$ and $\alpha_5 < 0$.

The gauge boson potential (24) mimics the standard Higgs potential (2). This becomes obvious after replacing the standard Higgs field $\Phi_0^{\dagger}\Phi_0$ by the composite Higgs field $-\mathrm{tr}[(\mathbf{V}_0\mathbf{V}_0)]$ via (8)-(10):

(26) $V_{\Phi} = \mu^2 \cdot \mathrm{tr}[(\mathbf{V}_0\mathbf{V}_0)] + \lambda \cdot \{\mathrm{tr}[(\mathbf{V}_0\mathbf{V}_0)]\}^2$

The comparison with (24) establishes a simple connection between the Higgs potential parameters μ^2, λ and the gauge boson couplings $-\alpha_0$, $-\alpha_5$:

(27) $\boxed{\mu^2 = -\alpha_0 \cdot \tfrac{1}{4} v^2}$ $\quad \tfrac{1}{4} v^2 \approx M_H^2 \quad$ $\boxed{\lambda = -\alpha_5}$ $\quad \alpha_0 < 0\ ,\ \alpha_5 < 0$

As a consequence of this similarity the minimization of the gauge boson potential becomes similar to that of the scalar Higgs potential. It requires only the solution of a quadratic equation in the variable $\Sigma_i(W_0^i W_0^i)$. The potential takes its minimum over a plane in the three-dimensional space spanned by the coordinates $(W_0^i W_0^i)$, i = 1,2,3:

(28) $V^{dyn} = 2^{-6} v^4 \cdot \alpha_0^2 / \alpha_5$ $\quad \boxed{\Sigma_i (\langle W_0^i \rangle \langle W_0^i \rangle) = -\tfrac{1}{4} v^2 / g^2 \cdot \alpha_0 / \alpha_5}$ $\quad$ at the minimum

This relation connects the EVs of the gauge bosons with the VEV v of the Higgs boson. Assuming equal EVs simplifies $\Sigma_i (\langle W_0^i \rangle \langle W_0^i \rangle)$ to $-3w^2$. This assumption will not be made here to allow for trade-offs between the three EVs $\langle W_0^i \rangle$ allowed by (28).

5. Compatibility Criteria and their Consequences

The EVs $\langle W_0^i \rangle$ obtained from the gauge boson potential are connected to the Higgs VEV v by the relations (11),(20) derived from the definition (10):

(29) $\Sigma_i (\langle W_0^i \rangle \langle W_0^i \rangle) = -v^2 / g^2$ General gauge

(30) $\Sigma_i (\langle W_0^i \rangle W^i) \approx -v / g^2 \cdot H$ General gauge

Compatibility between the two VEVs obtained in (28) and (29) from the two potentials fixes the ratio of the coupling constants in the gauge boson potential:

(31) $\boxed{\alpha_5 / \alpha_0 \approx \tfrac{1}{4}}$

This line of reasoning can be applied to other quantities as well. To obtain a second constraint for α_0, α_5 we apply this criterion to the gauge boson mass. In the standard model one obtains $M_W^2 = \tfrac{1}{4} g^2 v^2$ via the gauge-invariant derivatives D_μ in the kinetic Lagrangian $(D_\mu \Phi_0)^\dagger \cdot (D^\mu \Phi_0)$ of the Higgs boson. This term combines a pair of gauge bosons from D_μ with a pair of Higgs VEVs v from Φ_0. Attempting a similar scheme with the kinetic Lagrangian of the gauge bosons in (A1),(A2) would not work, since this term vanishes for the gauge bosons that form the composite Higgs boson (see Appendix A). Instead one can use the scheme that determines the Higgs mass in the standard model. After converting the Higgs potential from the lagrangian field H_0 to the observable field H, its mass is extracted from the H^2 term. Here we convert the gauge boson potential (25) from W_0^i to W^i via (19) and collect the mass terms $(W^i W^i)$:

(32) $(W_0^i W_0^i) \rightarrow (\langle W_0^i \rangle \langle W_0^i \rangle) + 2(\langle W_0^i \rangle W^i) + (W^i W^i)$ Unitary gauge

$$V_M^{dyn} \to -\alpha_0 \cdot \tfrac{1}{8} g^2 v^2 \cdot \Sigma_i (W^i W^i) - \alpha_5 \cdot \tfrac{1}{2} g^4 \cdot \{ \Sigma_i (\langle W_0^i \rangle \langle W_0^i \rangle) \cdot \Sigma_i (W^i W^i) + 2 [\Sigma_i (\langle W_0^i \rangle W^i)]^2 \}$$

$$\approx \{ -\alpha_0 \cdot \tfrac{1}{8} g^2 v^2 \cdot \Sigma_i (W^i W^i) - \alpha_5 \cdot \tfrac{1}{2} g^4 \cdot [-v^2/g^2 \cdot \Sigma_i (W^i W^i) + 2 v^2/g^4 \cdot H^2] \}$$

$$\approx \underbrace{(\tfrac{1}{4}\alpha_0 - 3\alpha_5)}\, g^2 v^2 \cdot \tfrac{1}{2} \Sigma_i (W^i W^i)$$

(33) $$= -M_W^2 \cdot \tfrac{1}{2} \Sigma_i (W^i W^i) \qquad M_W^2 = \tfrac{1}{4} g^2 v^2 \qquad \boxed{(\tfrac{1}{4}\alpha_0 - 3\alpha_5) \approx \tfrac{1}{4}}$$

The two sums containing $\langle W_0^i \rangle$ are converted to v^2, H^2 via (29),(30). H^2 is then converted to $-g^2 \Sigma_i (W^i W^i)$ via (14). The results for M_W^2 from the gauge and Higgs boson potentials are compared in (33). The resulting constraint for α_0, α_5 is combined with the constraint (31) from the VEVs to obtain the coupling constants of the gauge boson potential:

(34) $$\boxed{\alpha_0 \approx -\tfrac{1}{2} \qquad \alpha_5 \approx -\tfrac{1}{8}}$$

An evaluation of the diagrams in Fig. 1 can then be used to check the potential (25). The two Lagrangians L_0, L_5 are expected to provide the dominant contribution due to their close relation with the standard Higgs potential. But there are additional gauge-invariant Lagrangians available, such as analogs of L_0, L_5 that violate custodial symmetry [5].

A promising additional result arises when applying Fermi's Golden Rule to the relation (13) between observable gauge bosons and the composite Higgs boson. This involves averaging over all 3 polarization states ε_α of the 3 observable gauge fields W^i and summing over the 4 components of the scalar Higgs field Φ. By simply counting the degrees of freedom one obtains:

(35) $$4 = g^2 \cdot 3 \cdot 3 \qquad g^2 = 4/9 \qquad \boxed{g = 2/3}$$

That reproduces the value of the weak SU(2) coupling g with tree-level accuracy [3].

6. Summary and Outlook

In summary, a symmetry-breaking mechanism for the minimal SU(2) Yang-Mills theory is explored, where the gauge bosons themselves break the gauge symmetry via their quadratic and quartic self-interactions. The respective coupling constants α_0, α_5 can be mapped onto the two parameters μ, λ of the standard Higgs potential via the composite Higgs model proposed in [2]. An estimate of α_0 and α_5 is obtained by requiring that the scalar Higgs potential should not depend on whether it is derived from the gauge bosons or from the standard Higgs boson. The coupling constants α_0, α_5 can also be calculated in leading order from one-loop diagrams, thereby providing a self-consistency criterion.

This minimal model may serve as prototype for solving the mass gap problem for Yang-Mills gauge theories in general [4]. It suggests that gauge bosons are able to break their symmetry dynamically and thereby acquire mass. A pair of composite Higgs bosons formed by pairs of SU(2) gauge bosons represents the simplest possible bound state in Yang-Mills theories. This is the prototype for the glueballs that have been studied extensively for the SU(3) symmetry of the strong interaction [14]. The weaker SU(2) coupling and its smaller group size should make the mass gap problem more tractable. Calculations of SU(2) gauge boson self-interactions were reported in Refs. [5],[7]-[13], as discussed in [2]. Most of them focused on the high energy limit in order to explore unitarity constraints at the TeV scale. Experimentally, it will be interesting to get access to the threshold $v \approx 2M_{\mathrm{H}}$ for producing Higgs pairs [15], the SU(2) version of glueballs.

The composite Higgs model can be extended from SU(2) to the full symmetry group of the standard model by adding three rules for constructing Feynman diagrams:

1) Omit all diagrams containing the standard Higgs boson.
2) Define the composite Higgs boson in terms of SU(2) gauge bosons using (1),(17),(20). Treat this composite like a hadron composed of gluons.
3) In diagrams containing SU(2) gauge bosons, include their expectation values [16].

The extension to the SU(2)×U(1)$_{\mathrm{Y}}$ symmetry of the electroweak interaction is outlined in Appendix B. The addition of the strong interaction with SU(3) symmetry follows the standard model, since it does not involve the SU(2) gauge bosons.

Appendix A: Biquadratic Gauge Boson Lagrangians

While the potential $-(L_0+L_5)$ can be minimized analogous to the standard Higgs potential, the biquadratic Lagrangian L_{45} in (23) introduces scalar products $(\mathrm{W}_0^{\mathrm{j}}\mathrm{W}_0^{\mathrm{k}})$ with $\mathrm{j}\neq\mathrm{k}$ which tend to produce more complex potential surfaces. L_{45} is proportional to the non-abelian part L_{bq} of the kinetic gauge boson Lagrangian:

(A1) $$L_{\mathrm{kin}} = -\tfrac{1}{4}\,\Sigma_{\mathrm{i}}\,\mathrm{W}_{0\mu\nu}^{\mathrm{i}}\mathrm{W}_0^{\mathrm{i}\,\mu\nu} \qquad \mathrm{W}_{0\mu\nu}^{\mathrm{i}} = [\partial_\mu \mathrm{W}_{0\nu}^{\mathrm{i}} - \partial_\nu \mathrm{W}_{0\mu}^{\mathrm{i}}] - g\cdot\Sigma_{\mathrm{jk}}\,\varepsilon^{\mathrm{ijk}}\,\mathrm{W}_{0\mu}^{\mathrm{j}}\mathrm{W}_{0\nu}^{\mathrm{k}}$$

(A2) $$\begin{aligned} L_{\mathrm{bq}} &= -\tfrac{1}{4}g^2\cdot\Sigma_{\mathrm{ijkmn}}\,\varepsilon^{\mathrm{ijk}}\varepsilon^{\mathrm{imn}}\,\mathrm{W}_{0\mu}^{\mathrm{j}}\mathrm{W}_{0\nu}^{\mathrm{k}}\cdot\mathrm{W}_0^{\mathrm{m}\mu}\mathrm{W}_0^{\mathrm{n}\nu} \\ &= -\tfrac{1}{4}g^2\cdot\Sigma_{\mathrm{jkmn}}\,[\delta^{\mathrm{jm}}\delta^{\mathrm{kn}} - \delta^{\mathrm{jn}}\delta^{\mathrm{km}}]\,\mathrm{W}_{0\mu}^{\mathrm{j}}\mathrm{W}_{0\nu}^{\mathrm{k}}\cdot\mathrm{W}_0^{\mathrm{m}\mu}\mathrm{W}_0^{\mathrm{n}\nu} \\ &= -\tfrac{1}{4}g^2\cdot\Sigma_{\mathrm{j}\neq\mathrm{k}}\,[(\mathrm{W}_0^{\mathrm{j}}\mathrm{W}_0^{\mathrm{j}})\cdot(\mathrm{W}_0^{\mathrm{k}}\mathrm{W}_0^{\mathrm{k}}) - (\mathrm{W}_0^{\mathrm{j}}\mathrm{W}_0^{\mathrm{k}})^2\,] \end{aligned} \qquad \mathrm{i,j,k,m,n}=1,2,3$$

In the following we consider only gauge bosons that contribute to the composite Higgs boson (and to its potential). These are restricted by (17) to have a common polarization vector ε_α. The lagrangian gauge bosons in the last line of (A2) can then be decomposed into EVs and observable gauge bosons via (17): $W^i_0 = \langle W^i_0\rangle + W^i_T$, $\langle W^i_0\rangle = w\cdot\varepsilon_\alpha$, $W^i_T = w^i\cdot\varepsilon_\alpha$
For Higgs-like terms of the form (W^kW^k) in (3) one finds:

(A3) $-\frac{1}{4}g^2\cdot\Sigma_{j\neq k}[(\langle W^j_0\rangle\langle W^j_0\rangle)\cdot(W^kW^k) + (\langle W^k_0\rangle\langle W^k_0\rangle)\cdot(W^jW^j)] =$ 1st term

$= +\frac{1}{4}g^2w^2\cdot\Sigma_{j\neq k}[(W^kW^k)+(W^jW^j)] = +g^2w^2\cdot\Sigma_i(W^iW^i)$

(A4) $+\frac{1}{4}g^2\cdot\Sigma_{j\neq k}[(\langle W^j_0\rangle W^k)^2 + (W^j\langle W^k_0\rangle)^2] = +\frac{1}{4}g^2w^2\cdot\Sigma_{j\neq k}[(w^k)^2+(w^j)^2] =$ 2nd term

$= -\frac{1}{4}g^2w^2\cdot\Sigma_{j\neq k}[(W^kW^k)+(W^jW^j)] = -g^2w^2\cdot\Sigma_i(W^iW^i)$

In the 2nd term the vectors $\langle W^{j,k}_0\rangle$ or $W^{j,k}$ are first decomposed into $w\cdot\varepsilon_\alpha$ or $w^{j,k}\cdot\varepsilon_\alpha$, then sorted into equal pairs $(w^i)^2$, and eventually converted back into scalar products (W^iW^i). In the last conversion of (A3),(A4) the factor ¼ is compensated by reducing the 12 elements in the sum $\Sigma_{j\neq k}$ to 3 in the sum Σ_i. The final results for (A3) and (A4) cancel each other. Such cancellations also occur for the other terms generated by (A2). This is due to the requirement of equal polarization vectors for the gauge bosons forming the composite Higgs boson. That makes it possible to convert the 2nd term to the same form as the 1st term, but with opposite sign. Without this constraint one obtains non-zero mass-like terms with the polarizations $\alpha=\beta$, $\gamma=\delta$ for the 1st term and $\alpha=\gamma$, $\beta=\delta$ for the 2nd term (with $\alpha,\beta,\gamma,\delta$ referring to the gauge bosons j,j,k,k). The Lagrangian L_{45} defined in (25) is proportional to L_{bq} and thus exhibits the same properties.

In addition to the biquadratic vertex (A2) there are other diagrams of $O(g^2)$ to be considered, as shown in Figure 2. They consist of two trilinear vertices connected by a propagator (compare gauge boson scattering [7],[8]).

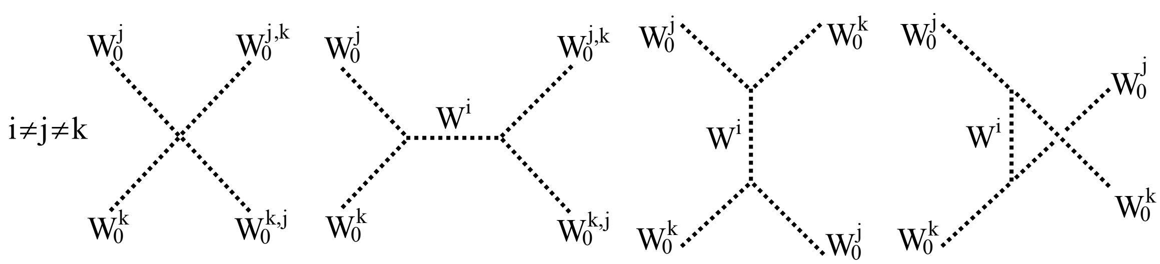

Figure 2 Biquadratic tree-level interactions of $O(g^2)$ for SU(2) gauge bosons W^j_0, W^k_0 (j≠k). In addition to the quadruple vertex there are diagrams consisting of trilinear vertices connected by an internal W^i. Time is to the right.

These vertices originate from mixed products between the derivatives and the non-abelian term in the Lagrangian (A1):

(A5) $\boxed{L_{\text{tri}} = \tfrac{1}{2} g \cdot \Sigma_{ijk}\, \varepsilon^{ijk}\, [\partial_\mu W^i_{0\nu} - \partial_\nu W^i_{0\mu}] \cdot W^j_{0\mu} W^k_{0\nu}}$

The derivatives associated with the triple vertices eliminate the EVs from the internal bosons W^i in Figures 1,2 while the external bosons W^j_0, W^k_0 keep their EVs.

Appendix B: Extension to the SU(2)×U(1)$_Y$ Electroweak Symmetry

The electroweak interaction mixes the SU(2) gauge boson W^3_0 with the U(1)$_Y$ gauge boson B_0 to form the new mass eigenstates Z_0 and A_0 (the photon). The remaining SU(2) gauge bosons W^1_0, W^2_0 form the charge eigenstates $W^\pm_0$:

(B1) $Z_0 = (g W^3_0 - g' B_0)/(g^2+g'^2)^{1/2}$ $\qquad$ $A_0 = (g B_0 + g' W^3_0)/(g^2+g'^2)^{1/2}$

$W^+_0 = (W^1_0 - i W^2_0)/\sqrt{2}$ $\qquad$ $W^-_0 = (W^1_0 + i W^2_0)/\sqrt{2}$

The ratio $g'/g = \tan\theta_w$ determines the weak mixing angle θ_w. The couplings g, g', and e of the symmetry groups SU(2), U(1)$_Y$, and U(1)$_{EM}$ are given by:

$g/(g^2+g'^2)^{1/2} = \cos\theta_w = c_w$ $\quad$ $g'/(g^2+g'^2)^{1/2} = \sin\theta_w = s_w$ $\quad$ $e = gg'/(g^2+g'^2)^{1/2} = g \cdot \sin\theta_w = g' \cdot \cos\theta_w$

Analogous to (17) one can extract the EVs from the gauge bosons Z_0 and B_0:

(B2) $Z_0 = \langle Z_0\rangle + Z_T \quad \langle Z_0\rangle = z \cdot \varepsilon_\alpha \quad \langle Z_T\rangle = 0 \quad Z_T = z \cdot \varepsilon_\alpha$ $\qquad$ Landau gauge

$B_0 = \langle B_0\rangle + B_T \quad \langle B_0\rangle = b \cdot \varepsilon_\alpha \quad \langle B_T\rangle = 0 \quad B_T = b \cdot \varepsilon_\alpha$

The EV of the photon A_0 vanishes, since it represents the remaining electromagnetic U(1)$_{EM}$ symmetry. The VEVs z, b are obtained by inserting $\langle W^3_0\rangle = w^3 \cdot \varepsilon_\alpha$ from (17) into the EV of (B1), taking into account $\langle A_0\rangle = 0$:

(B3) $\langle B_0\rangle = -g'/g \cdot \langle W^3_0\rangle = -g'/g \cdot w^3 \cdot \varepsilon_\alpha$ $\qquad$ $b = -w^3 \cdot g'/g = -w^3 \cdot s_w/c_w$

$\langle Z_0\rangle = (g^2+g'^2)^{1/2}/g \cdot \langle W^3_0\rangle = z \cdot \varepsilon_\alpha$ $\qquad$ $\boxed{z = w^3 \cdot (g^2+g'^2)^{1/2}/g = w^3/c_w}$

The ratio of the VEVs $w^3/z = \cos\theta_w$ is identical to the tree-level mass ratio M_W/M_Z.

In order to generalize the definitions of the composite Higgs boson and the symmetry-breaking gauge boson potential to SU(2)×U(1)$_Y$ one has to include the U(1)$_Y$ gauge boson B_0 in the gauge-invariant derivatives (5),(A1):

(B4) $D_\mu \mathbf{U} = \partial_\mu \mathbf{U} - i g \cdot \mathbf{W}_{0,\mu} \mathbf{U} + i g' \mathbf{U} \mathbf{B}_{0,\mu}$ $\qquad$ $\mathbf{W}_{0,\mu} = \Sigma_j W^j_{0,\mu} \cdot \tfrac{1}{2} \boldsymbol{\tau}^j$ $\qquad$ $\mathbf{B}_{0,\mu} = B_{0,\mu} \cdot \tfrac{1}{2} \boldsymbol{\tau}^3$

(B5) $W^i_{0\mu\nu} = [\partial_\mu W^i_{0\nu} - \partial_\nu W^i_{0\mu}] - g \cdot \Sigma_{jk}\, \varepsilon^{ijk}\, W^j_{0\mu} W^k_{0\nu}$ $\qquad$ $B_{0\mu\nu} = [\partial_\mu B_{0\nu} - \partial_\nu B_{0\mu}]$

From there one can proceed as in Sections 2,3 after the following conversions:

(B6) $W_0^1 \to (W_0^+ + W_0^-)/\sqrt{2}$ $\qquad W_0^2 \to i\,(W_0^+ - W_0^-)/\sqrt{2}$

$W_0^3 \to (g Z_0 + g' A_0)/(g^2 + g'^2)^{1/2} = c_w Z_0 + s_w A_0$

The definition (10) of the composite Higgs boson and the relations (11),(20),(14) become:

(B7) $\boxed{H_0^2 = -g^2 \cdot [2(W_0^+ W_0^-) + (Z_0 Z_0)/c_w^2]}$ Unitary gauge

(B8) $\boxed{v^2 = -g^2 \cdot [2(\langle W_0^+\rangle\langle W_0^-\rangle) + (\langle Z_0\rangle\langle Z_0\rangle)/c_w^2]}$ General gauge

(B9) $\boxed{v\,H \approx -g^2 \cdot [(\langle W_0^+\rangle W^-) + (W^+\langle W_0^-\rangle) + (\langle Z_0\rangle Z_0)/c_w^2]}$ General gauge

(B10) $\boxed{H^2 \approx -g^2 \cdot [2(W^+ W^-) + (ZZ)/c_w^2]}$ Unitary gauge

The gauge boson potential takes again the form $V^{dyn} = -L^{dyn} = -L_0 - L_5$:

(B11) $\boxed{L_0 = \alpha_0 \cdot M_W^2 \cdot [(W_0^+ W_0^-) + ½(Z_0 Z_0)/c_w^2]}$ $\qquad M_W^2 = ¼\, g^2 v^2$

$\boxed{L_5 = \alpha_5 \cdot ¼ g^4 \cdot [(W_0^+ W_0^-)^2 + (W_0^+ W_0^-)(Z_0 Z_0)/c_w^2 + ¼(Z_0 Z_0)^2/c_w^4]}$

The photon is massless and therefore does not contribute to L_0, L_5 which are built from mass Lagrangians. The potential minimum has the same depth as in (28) for pure SU(2):

(B12) $V^{dyn} = 2^{-6} v^4 \cdot \alpha_0^2/\alpha_5$ for $[2(\langle W_0^+\rangle\langle W_0^-\rangle) + (\langle Z_0\rangle\langle Z_0\rangle)/c_w^2] = -¼\, v^2/g^2 \cdot \alpha_0/\alpha_5$

The minimum extends now along a line in the two-dimensional space spanned by the coordinates $(W_0^+ W_0^-)$ and $(Z_0 Z_0)$, as shown in Fig. 7a of [2]. A well-defined point on this line can be selected by requiring identical VEVs w^i for the three gauge bosons W_0^i, even after mixing. This converts (B8) into $v^2 = g^2 [2w^2 + z^2/c_w^2]$ and (B3) into $z^2 = w^2/c_w^2$, with the common VEV $w^i = w = v/[g(2 + c_w^{-4})^{1/2}] = 194$ GeV and $z = 220$ GeV. The ratio of the VEVs $w/z = \cos\theta_w$ then becomes equal to the tree-level mass ratio M_W/M_Z.

The compatibility criteria (31),(33) for the coupling constants α_0, α_5 require identical results for gauge boson EVs and masses from either the Higgs potential (2) or the gauge boson potential (B11). The comparison of the EVs in (B8) and (B12) yields:

(B13) $\boxed{\alpha_5/\alpha_0 = ¼}$

This holds for all values of g'. Consistency of the gauge boson masses involves the quadratic part of the potential for observable gauge bosons. As in (32),(33) one obtains:

(B14) $(W_0^+ W_0^-) \to (\langle W_0^+\rangle\langle W_0^-\rangle) + [(\langle W_0^+\rangle W^-) + (W^+\langle W_0^-\rangle)] + (W^+ W^-)$

$(Z_0 Z_0) \to (\langle Z_0\rangle\langle Z_0\rangle) + 2(\langle Z_0\rangle Z_0) + (ZZ)$

$$V_M^{\rm dyn} \to -\alpha_0 \cdot \tfrac{1}{8} g^2 v^2 \cdot [2(W^+W^-) + (ZZ)/c_w^2] \qquad \text{Unitary gauge}$$
$$-\alpha_5 \cdot \tfrac{1}{2} g^4 \cdot \{ [2(\langle W_0^+\rangle\langle W_0^-\rangle) + (\langle Z_0\rangle\langle Z_0\rangle)/c_w^2] \cdot [2(W^+W^-) + (ZZ)/c_w^2]$$
$$+ 2[(\langle W_0^+\rangle W^-) + (W^+\langle W_0^-\rangle) + (\langle Z_0\rangle Z_0)/c_w^2]^2 \}$$
$$\approx (\tfrac{1}{4}\alpha_0 - 3\alpha_5) \cdot g^2 v^2 \cdot [(W^+W^-) + \tfrac{1}{2}(ZZ)/c_w^2] \;=\; -[M_W^2 \cdot (W^+W^-) + \tfrac{1}{2} M_Z^2 \cdot (ZZ)]$$

Use of (B8)-(B10) leads to the last line. The compatibility condition becomes:

(B15) $M_W^2 = \tfrac{1}{4} g^2 v^2 \qquad M_Z^2 = M_W^2/c_w^2 \qquad \boxed{(\tfrac{1}{4}\alpha_0 - 3\alpha_5) \approx \tfrac{1}{4}}$

This is again independent of g', leaving the combined result (34) for α_0, α_5 unchanged:

(B16) $\boxed{\alpha_0 \approx -\tfrac{1}{2} \qquad \alpha_5 \approx -\tfrac{1}{8}}$

The full set of diagrams for the Lagrangians L_0, L_5 (including fermions) is given in Fig. 4 of [2]. Calculations related to the coupling constants α_0, α_5 have been published in [5],[7]-[13]. They still need to be performed for an energy scale comparable to the EVs $\{w,z\} \cdot \varepsilon_\alpha$ around which the fields W, Z oscillate. One can estimate the (transverse) self-energy Σ_T of the gauge bosons using the identification $L_0 \approx [\Sigma_T^W \cdot (W_0^+ W_0^-) + \Sigma_T^Z \cdot \tfrac{1}{2}(Z_0 Z_0)]$, i.e., replacing $\{M_W^2, M_W^2/c_w^2\}$ by $\{\Sigma_T^W, \Sigma_T^Z\}$ in (B11). Assuming a common VEV $w^i = w$ we can use the values of $\{w,z\}$ from above to obtain:

(B17) $\Sigma_T^W(w^2) \approx \Sigma_T^Z(z^2) \approx -\alpha_0 \cdot \tfrac{1}{4} g^2 v^2 \approx -(57.7\,\text{GeV})^2 \quad \text{at} \quad w = 194\,\text{GeV},\; z = 220\,\text{GeV}$

References

1. G. Cvetic, *Top-Quark Condensation*, Rev. Mod. Phys. **71**, 513 (1999); R. Contino, *The Higgs as a Composite Nambu-Goldstone Boson*, arXiv:1005.4269 [hep-ph] (2010); C. Csaki, C. Grojean, J. Terning, *Alternatives to an Elementary Higgs*, Rev. Mod. Phys. **88**, 045001 (2016).

2. F. J. Himpsel, *A Higgs Boson Composed of Gauge Bosons*, arXiv:1502.06438 [hep-ph] (2015). For follow-up papers see Ref. [17] below.

3. Experimental values are from: *2017 Review of Particle Physics*, C. Patrignani et al. (Particle Data Group), Chin. Phys. C **40**, 100001 (2016) and 2017 updates. We use: $G_F = 1.166379 \cdot 10^{-5}\,\text{GeV}^{-2}$, $M_W = 80.385\,\text{GeV}$, $M_Z = 91.1876\,\text{GeV}$, $M_H = 125.09\,\text{GeV}$, $\alpha = e^2/4\pi = (137.036)^{-1}$, $\alpha = (127.950)^{-1}$ at M_Z, $g^2 = e^2/\sin^2\theta_w = 4\pi\alpha/\sin^2\theta_w$. For $\sin^2\theta_w$ there exist various definitions which differ in higher orders of perturbation theory. Two common schemes are $\overline{\text{MS}}$ with $\sin^2\theta_w = 0.2313$, $g^2 = 0.425$, $g = 0.652$ at M_Z and on-shell with $\sin^2\theta_w = 0.2234$, $g^2 = 0.440$, $g = 0.663$.

4. C. N. Yang, R. L. Mills, *Conservation of Isotopic Spin and Isotopic Gauge Invariance*, Phys. Rev. B **96**, 191 (1954); L. D. Fadeev, V. N. Popov, *Feynman Diagrams for the Yang-Mills Field*, Phys. Lett. B **25**, 29 (1967); G. t'Hooft, M. Veltman, *Regularization and Renormalization of Gauge Fields*, Nucl. Phys. B **44**, 189 (1972); A. Mann, *Classical Solutions of SU(2) Yang-Mills Theories*, Rev. Mod. Phys. **51**, 461 (1979); N. Seiberg, E. Witten, *Electric-Magnetic Duality, Monopole Condensation, and Confinement in N=2 Supersymmetric Yang-Mills Theory*, Nucl. Phys. B **426**, 19 (1994). One of the seven Millenium Prizes of the Clay Mathematics Institute is dedicated to a rigorous existence proof of Yang-Mills gauge theories, including the presence of a mass gap, i.e., a nonzero mass of the lightest particle.

5. D. Espriu, B. Yencho, *Longitudinal WW Scattering in Light of the "Higgs Boson" Discovery*, Phys. Rev. D **87**, 055017 (2013); R. Alonso, M. B. Gavela, L. Merlo, S. Rigolin, J. Yepes, *The Effective Chiral Lagrangian for a Light Dynamical "Higgs Particle"*, Phys. Lett. B **722**, 330 (2013); R. Contino, M. Ghezzi, C. Grojean, M. Mühlleitner, M. Spira, *Effective Lagrangian for a Light Higgs-like Scalar*, JHEP **07**, (2013) 035; R. L. Delgado, A. Dobado, F. J. Llanes-Estrada, *One-Loop $W_L W_L$* and *$Z_L Z_L$ Scattering from the Electro-weak Chiral Lagrangian with a Light Higgs-Like Scalar*, JHEP **02** (2014) 121; I. Brivio et al., *Disentangling a Dynamical Higgs*, JHEP **03** (2014) 024; G. Buchalla, O. Catà, and C. Krause, *Complete Electroweak Chiral Lagrangian with a light Higgs at NLO*, Nucl. Phys. B **880**, 552 (2014). The vector of 2×2 matrices $\mathbf{V}_{0,\mu}$ defined in Eq. (6) is hermitian (similar to Buchalla et al.). It is anti-hermitian in much of the earlier literature and in Versions 1-5 of this paper.

6. L. D. Landau, *On the Angular Momentum of a System of two Photons*, Dokl. Akad. Nauk SSSR **60**, 207 (1948); C. N. Yang, *Selection Rules for the Dematerialization of a Particle Into Two Photons*, Phys. Rev. **77**, 242 (1950).

7. B. W. Lee, C. Quigg, H. B. Thacker, *Weak Interactions at Very High Energies: The Role of the Higgs Boson Mass*, Phys. Rev. D **16**, 1519 (1977); E. Yehudai, *Probing W-γ Couplings Using $\gamma\gamma \to W^+W^-$*, Phys. Rev. D **55**, 3434 (1991).

8. A. Denner, S. Dittmaier, R. Schuster, *Radiative Corrections to $\gamma\gamma \to W^+W^-$ in the Electroweak Standard Model*, Nucl. Phys. B **452**, 80 (1995); A. Denner, S. Dittmaier, T. Hahn, *Radiative Corrections to ZZ→ZZ in the Electroweak Standard Model*, Phys. Rev. D **56**, 117 (1997); A. Denner, T. Hahn, *Radiative Corrections to $W^+W^- \to W^+W^-$ in the Electroweak Standard Model*, Nucl. Phys. B **525**, 27 (1998).

9. The definitions of L_4 and L_5 are widely agreed upon in the literature, but L_0 is not. We define L_0 equal to α_0 multiplied by the Lagrangian L_0 given in: E. Boos, H.-J. He, W. Kilian, A. Pukhov, C.-P. Yuan, P. M. Zerwas, *Strongly Interacting Vector Bosons at TeV $e^{\pm}e^-$ Linear Colliders*, Phys. Rev. D **57**, 1553 (1998).

10. M. Böhm, H. Spiesberger, and W. Hollik, *On the 1-Loop Renormalization of the Electroweak Standard Model and its Applications to Leptonic Processes*, Fortschr. Phys. **34**, 687 (1986), see Figures 10 and 11, Sections 5.2 and 6.1, Appendix B; W. Hollik, *Radiative Corrections in the Standard Model and their Role for Precision Tests of the Electroweak Theory*, Fortschr. Phys. **38**, 165 (1990), Eqs. (4.24),(4.27) and Appendix B.

11. A. Denner, *Techniques for the Calculation of Electroweak Corrections at the One-Loop Level and Results for W-Physics at LEP200*, Fortschr. Phys. **41**, 307 (1993) and arXiv:0709.1075 [hep-ph] (2007).

12. B. A. Kniehl, *Higgs Phenomenology at one Loop in the Standard Model*, Physics Reports **240**, 211 (1994), Appendix B.

13. D. Bardin and G. Passarino, *The Standard Model in the Making, Precision Study of the Electroweak Interactions*, Int. Series of Monographs in Physics Vol. 104, Clarendon Press, Oxford (1999), Sections 5.2.4, 5.2.5.

14. V. Mathieu, N. Kochelev, and V. Vento, *The Physics of Glueballs*, Int. J. Mod. Phys. E**18**, 1 (2009), arXiv:0810.4453 [hep-ph] (2008). For wave functions, see Equations (22)-(32); W. Ochs, *The Status of Glueballs*, J. Phys. G **40**, 043001 (2013).

15. B. Grinstein, M. Trott, *Higgs-Higgs Bound State due to New Physics at a TeV*, Phys. Rev. D **76**, 073002 (2007); J. Grigo, J. Hoff, K. Melnikov, M. Steinhauser, *On the Higgs Boson Pair Production at the LHC*, Nucl. Phys. B **875**, 1 (2013); F. Bishara, R. Contino, J. Rojo, *Higgs Pair Production in Vector Boson Fusion at the LHC and beyond*, Eur. Phys. J. C **77**, 481 (2017); Editors: B. D. Micco, M. Gouzevitch, J. Mazzitelli, C. Vernieri, *Higgs Boson Potential at Colliders: Status and Perspectives*, arXiv:1910.00012 [hep-ph] (2020). See also abstracts of talks about Higgs pair production, given at the 2022 April Meeting of the APS, Sessions B10 and L10, published in the Bulletin of the Americal Physical Society (2022).

16. Notice that the fermion couplings to the gauge boson EVs of the form $(\bar{\psi}^f\gamma^\mu\psi^f)\cdot\langle W^i_{0,\mu}\rangle$ vanish after averaging over all virtual gauge bosons W^i_0 in vacuum.

17. This is an updated version of arXiv:1801.04604 [hep-ph]. It can be downloaded at: https://pages.physics.wisc.edu/~himpsel/1801.04604update.pdf
Two follow-up papers on the Higgs couplings for this model can be downloaded at:
https://pages.physics.wisc.edu/~himpsel/FermionMass12.pdf
https://pages.physics.wisc.edu/~himpsel/BosonCoupling2.pdf