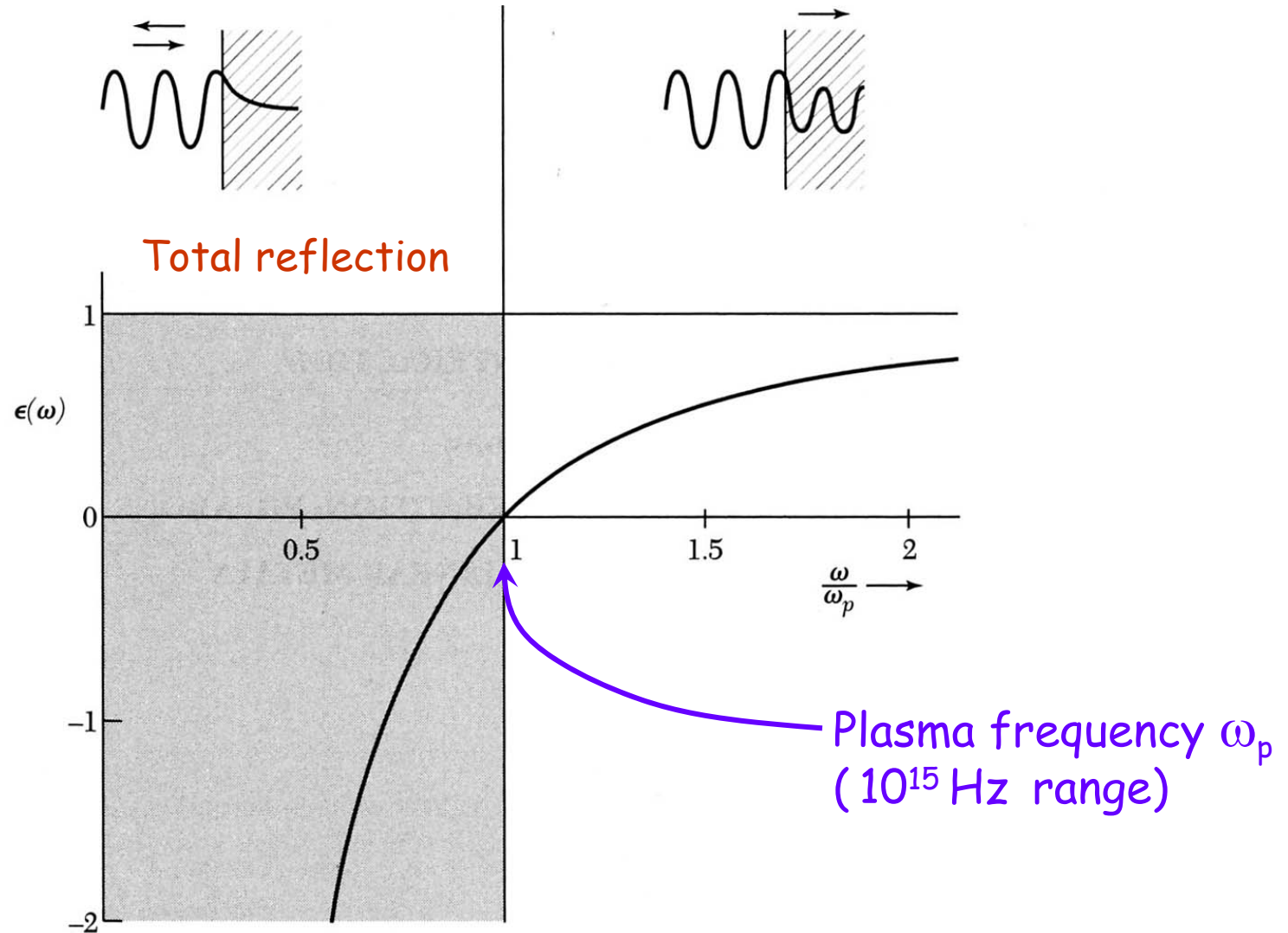


# The Dielectric Function of a Metal ("Jellium")



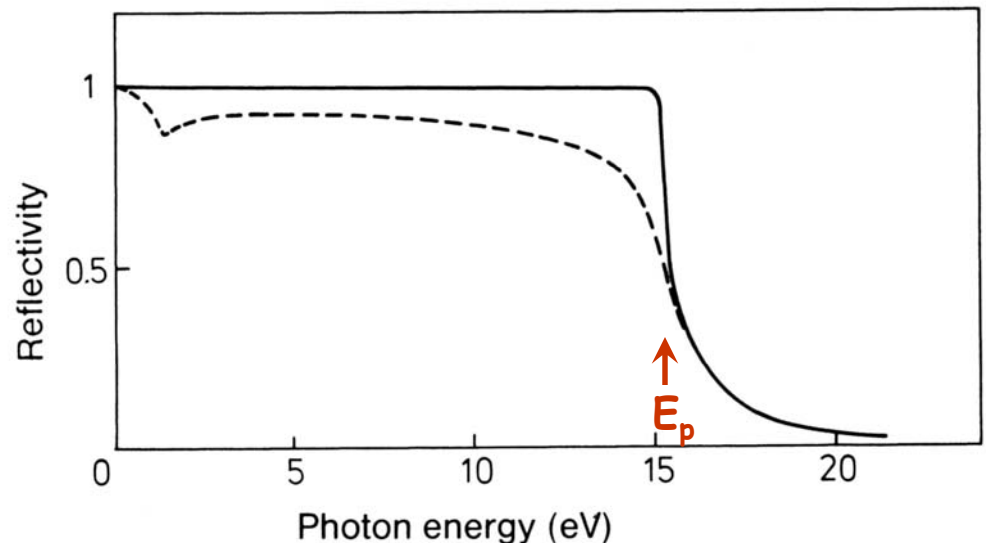
**Figure 1** Dielectric function  $\epsilon(\omega)$  or  $\epsilon(\omega, 0)$  of a free-electron gas versus frequency in units of the plasma frequency  $\omega_p$ . Electromagnetic waves propagate without damping only when  $\epsilon$  is positive and real. Electromagnetic waves are totally reflected from the medium when  $\epsilon$  is negative.

# Why are Metals Shiny ?

An electric field cannot exist inside a metal, because metal electrons follow the field until they have compensated it. An example is the image charge, which exactly cancels the field of any external charge. This is also true for an electromagnetic wave, where electrons respond to the changing external field. As a result, the electromagnetic wave cannot enter a metal and gets totally reflected in the region with  $\epsilon < 0$ .

Above the plasma frequency, however, the external field oscillates too fast for the electrons to follow. A metal loses its reflectivity. The corresponding photon energy is the **plasmon energy**  $E_p = \hbar\omega_p$ , typically 10-30 eV (deep into the ultraviolet).

The reflectivity of aluminum cuts off at its plasmon energy. Data (dashed) are compared to the jellium model (full). The difference is due to damping of the electrons in real metals.



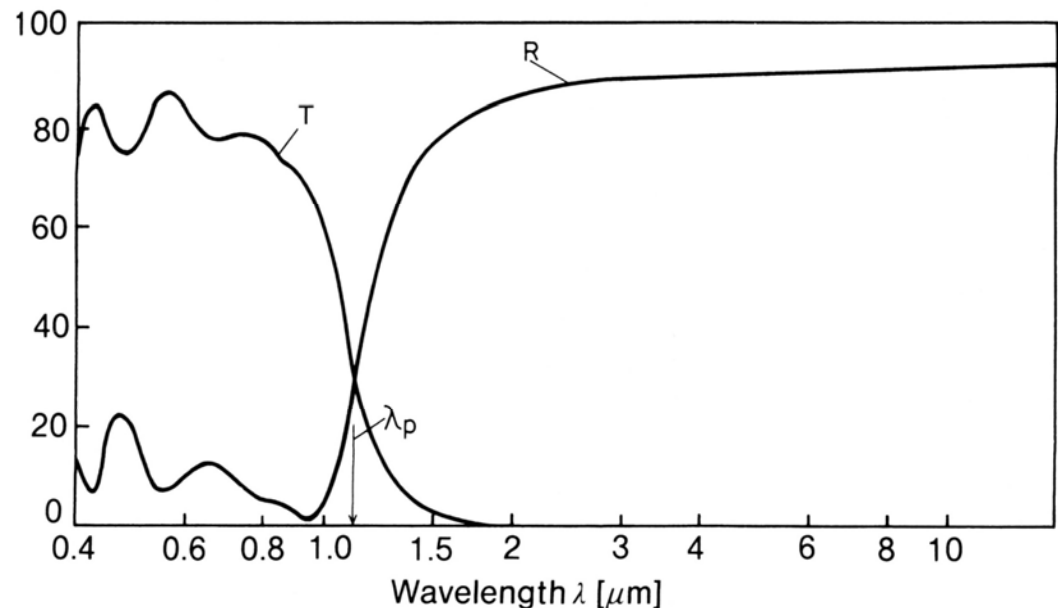
# Plasma Frequency and Energy-Saving Window Coatings

The reflectivity cutoff at the plasmon energy can be used for energy-saving window coatings which transmit visible sunlight (photon energy above  $E_p$ ), but reflect thermal IR radiation back into a heated room.

To get a reflectivity **cutoff in the infrared** one needs a smaller electron density than in a metal. A highly-doped semiconductor fits just right, such as **indium-tin-oxide (ITO)**. This material is also widely used as transparent front electrode for solar cells, LEDs, and liquid crystal displays.

An ITO film transmits visible light and reflects thermal infrared radiation, keeping the heat inside a building.

R = Reflectivity  
T = Transmission



Compared to the previous slide the the x-scale is inverted ( $\lambda$  instead of  $E$ , with  $\lambda \propto 1/E$ )

# What is a Plasmon ?

A plasmon is a **density wave in an electron gas**. It is analogous to a sound wave, which is a density wave in a gas consisting of molecules.

Plasmons exist mainly in **metals**, where electrons are weakly bound and free to roam. The **free electron gas (jellium)** model provides a good approximation.

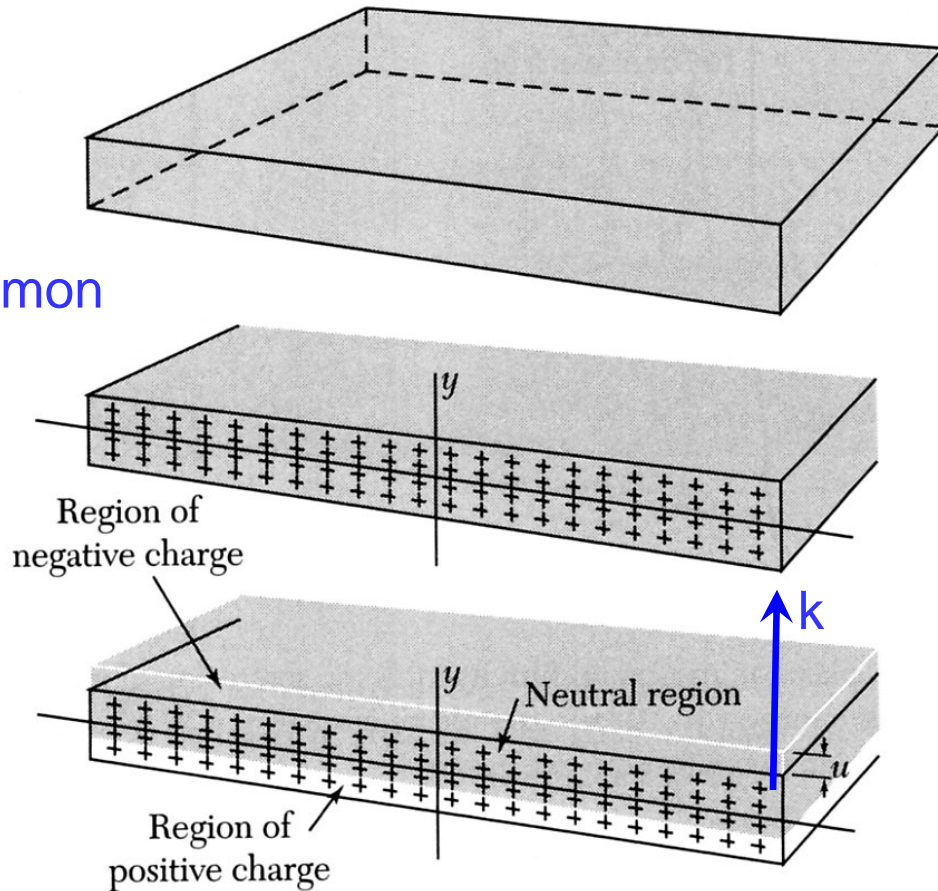
The electrons in a metal can wobble like a piece of jelly, pulled back by the attraction of the positive metal ions that they leave behind.

In contrast to the single electron wave function that we encountered already, a plasmon is a **collective wave**. Billions of electrons oscillate in sync.

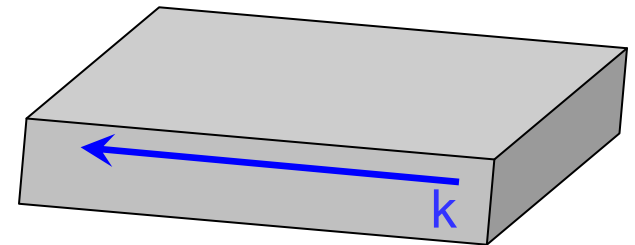
# The Plasmon Resonance

The electron gas has a resonance right at the **plasma frequency**  $\omega_p$ . This resonance frequency increases with the **electron density**  $n$ , since the electric restoring force is proportional to the displaced charge (analogous to the force constant  $f$  of a spring):  $\omega_p \propto \sqrt{n}$

Bulk Plasmon

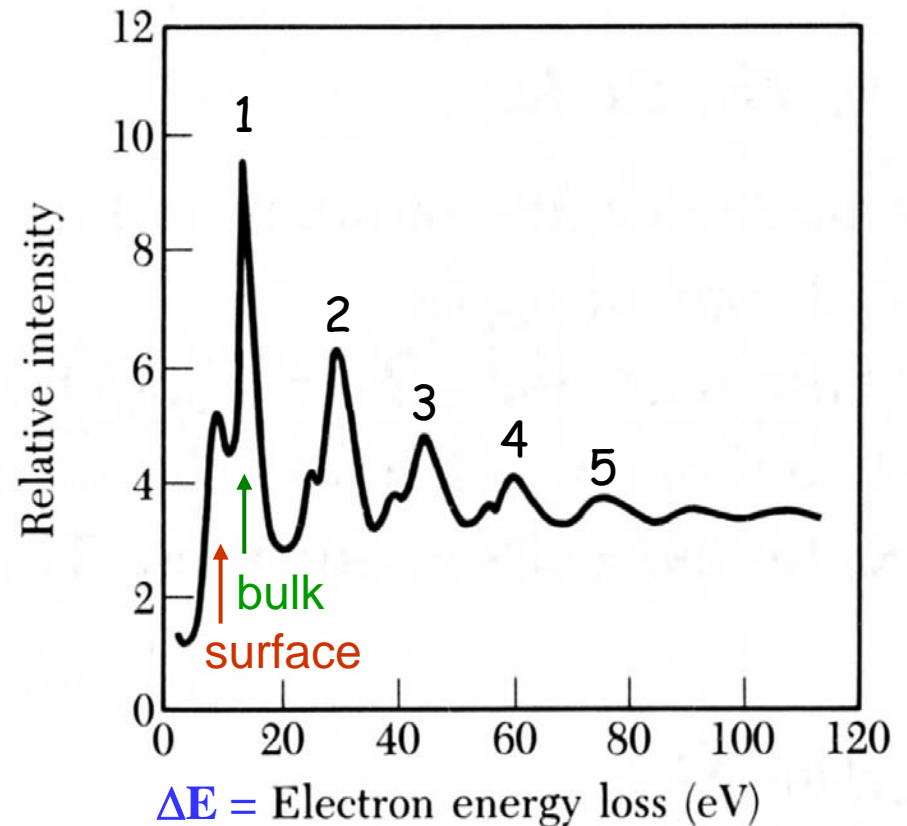
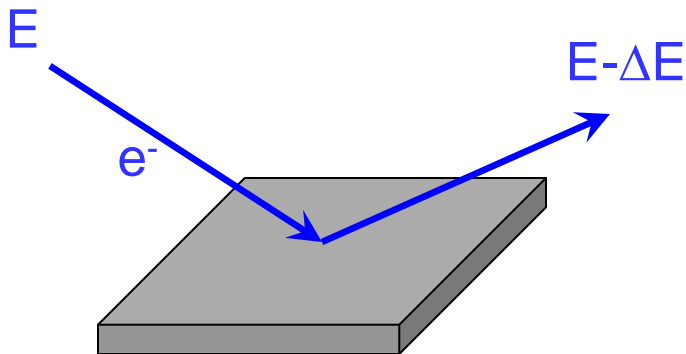


Surface Plasmon



# Observation of Plasmons

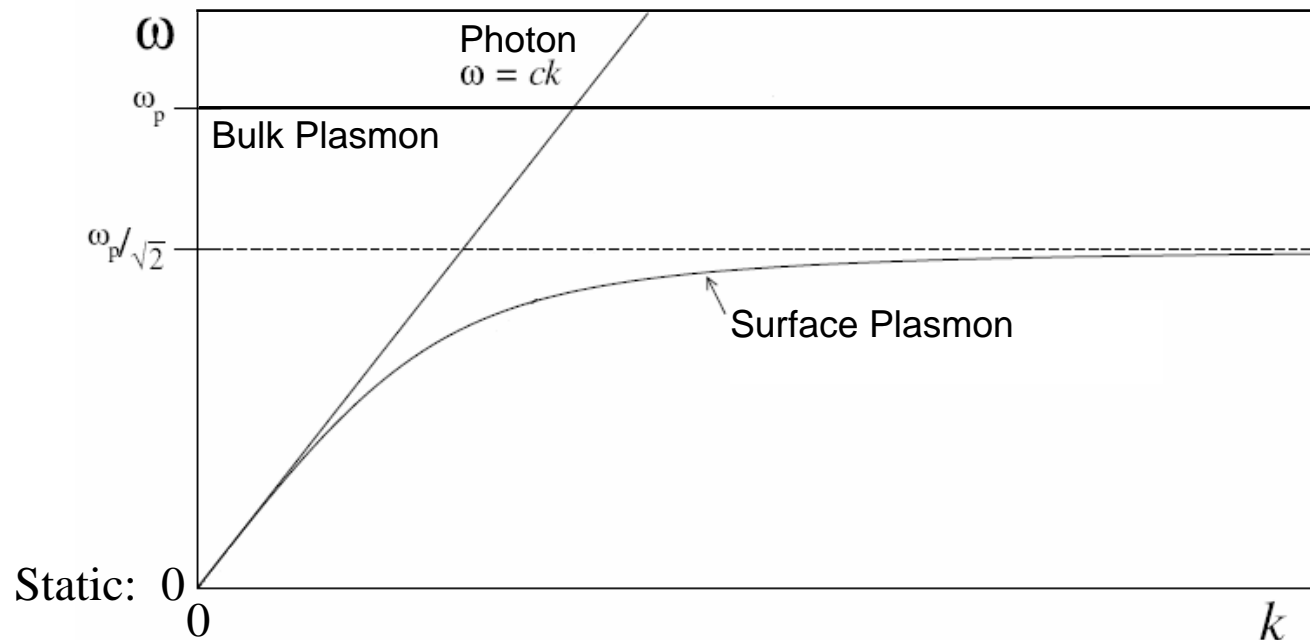
Plasmons can be observed by electron energy loss spectroscopy (EELS). Electrons with an energy of several keV are reflected from a metal surface and lose energy by exciting 1, 2, 3, ... plasmons. Large peaks at multiples of 15.3 eV are from bulk plasmons, and the smaller peaks at multiples of 10.3 eV from surface plasmons. The surface plasmon energy is  $\sqrt{2}$  times lower than the bulk plasmon energy.



# Quantum Numbers of Plasmons

Like any other particle/wave in a solid, a plasmon has **energy**  $E=\hbar\omega$  and **momentum**  $\mathbf{p}=\hbar\mathbf{k}$  as quantum numbers. The same  $E(\mathbf{k})$  plots can be used as for individual electrons and phonons.

The surface plasmon band joins the photon band at very small  $k$  and  $\omega$ . The limit  $\omega \rightarrow 0$  describes the **image charge**.





# Coupling of Light and Plasmons

To combine optoelectronics with plasmonics one has to convert light (photons) into plasmons. This is not as simple as it sounds.

**Bulk plasmons** are **longitudinal** oscillations (parallel to the propagation direction), while photons are transverse (perpendicular to the propagation). They don't match.

**Surface plasmons** are **transverse**, but they are mismatched to photons in their momentum. The two  $E(k)$  curves only coincide at  $k=0$ . It is possible to provide the necessary momentum  $\hbar k$  by a grating, which transfers the  $\Delta k = 2\pi/d$  ( $d$ =line spacing). Thereby one can control the wavelength of surface plasmons.



# Attenuated Total Reflection

Another method to couple photons and surface plasmons uses attenuated total reflection at a metal-coated glass surface. The exponentially damped (evanescent) light wave escaping from the glass can be matched to a surface plasmon (or thin film plasmon) in the metal coating. This technique is surface sensitive and is used for bio-sensors.

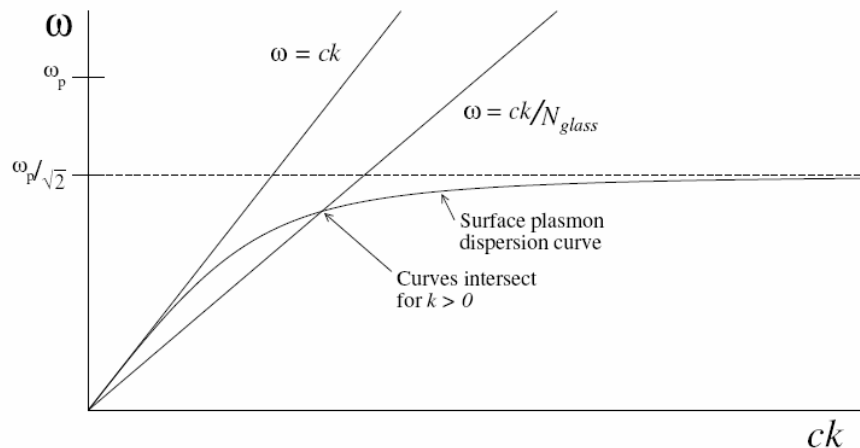
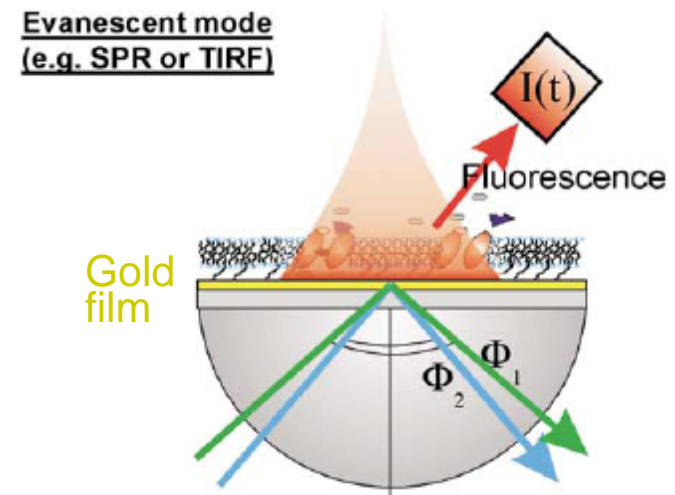


Figure 2: Dispersion curves for surface plasmons, air ( $\omega = ck$ ) and glass ( $\omega = ck/N_{glass}$ , assuming  $N_{glass}$  is independent of wavelength). Note that the curves do not intersect for air and plasmons, indicating that plasmons cannot be excited from light propagating from air to the metal surface.



# Low-Dimensional Plasmons in Nanostructures

The same way as the energy of single electron waves becomes **quantized by confinement in a nanostructure**, plasmons are affected by the boundary conditions in a thin film or a nanoparticle.

Plasmons in metal nanoparticles are often called **Mie-resonances**, after Gustav Mie who calculated them hundred years ago. Their resonance energy and color depend strongly on their size, similar to the color change induced in semiconductor nanoparticles by confinement of single electrons. In both cases, **smaller particles have higher resonance energy** (blue shift).

# Nanotechnology in Roman Times: The Lycurgus Cup

Plasmons of gold nanoparticles in glass reflect green, transmit red.

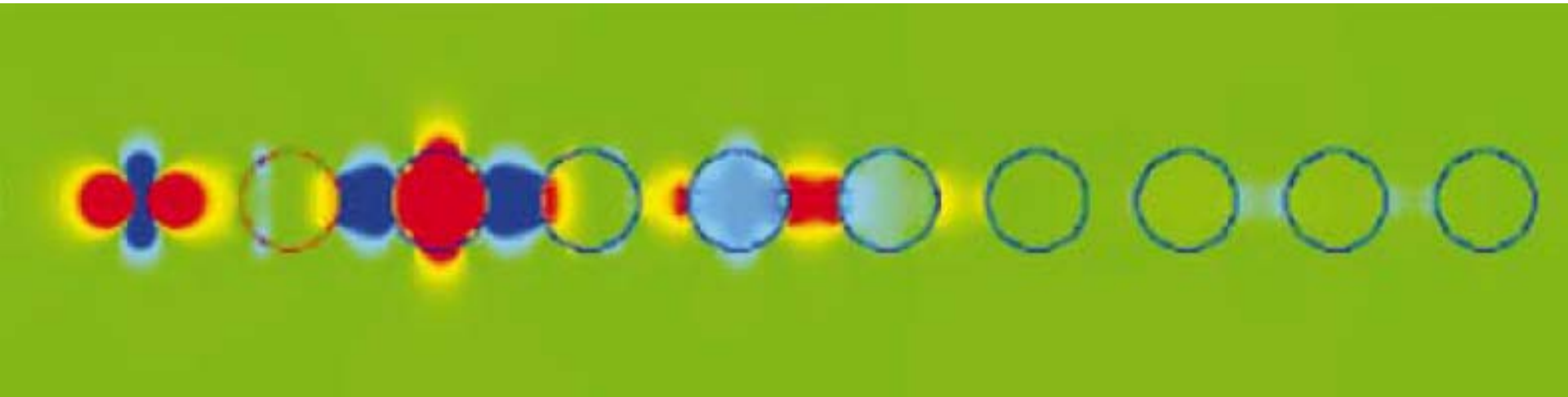


# Plasmonics

- Optoelectronics is much faster than regular electronics (waveguides, optical fibers).
- But the long wavelength of light ( $\approx \mu\text{m}$ ) creates a problem for **extending optoelectronics** into the nanometer regime.
- A possible way out is the conversion of light into **plasmons**.
- They have much **shorter wavelength** than light (larger  $k$ ).

# Plasmonic Waveguide ("Wire")

## Consisting of a String of Gold Nanoparticles



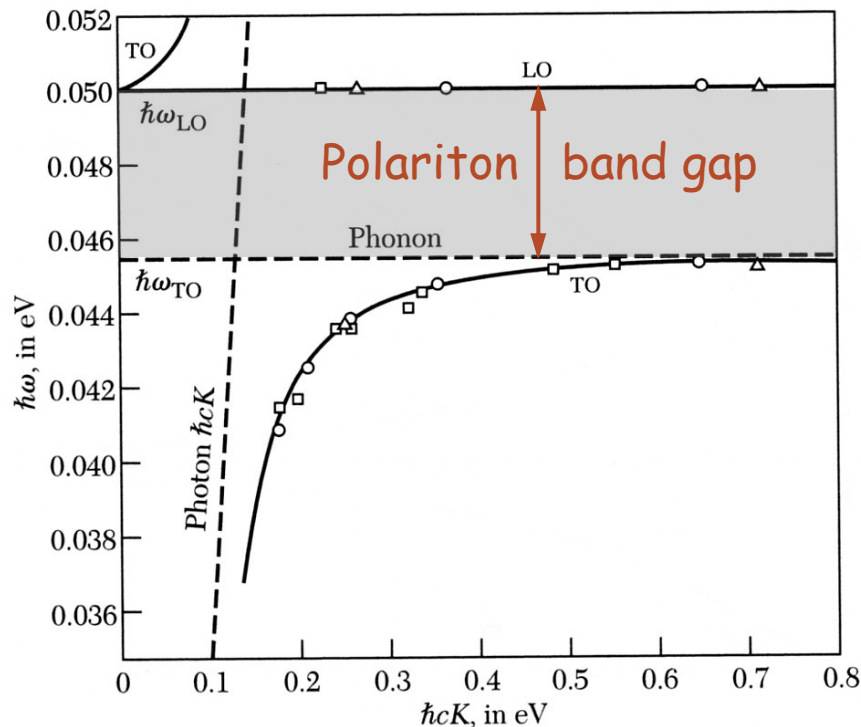
Calculated electric field of an electromagnetic pulse propagating along a string of gold nanoparticles

The energy is transferred between neighboring gold particles by **coupled Mie-resonances**.



# Polariton = Photon + Phonon

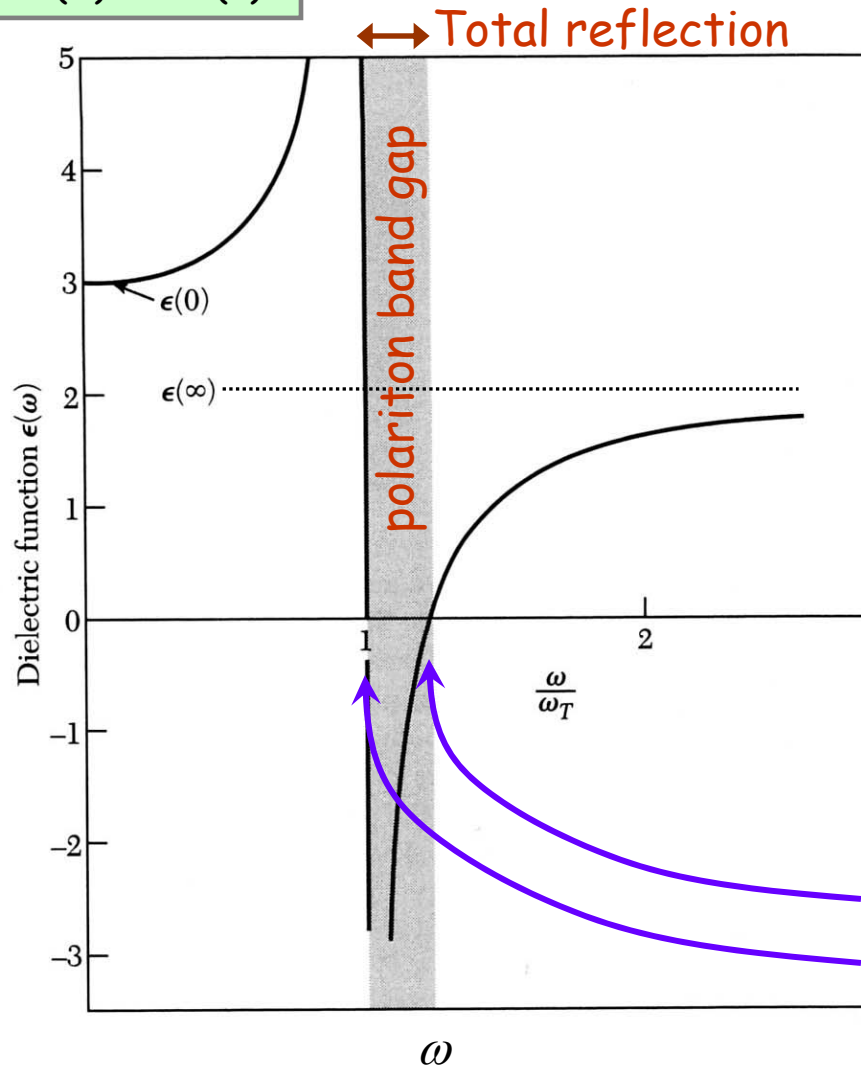
A polariton is an phonon that interacts with photons. Its  $E(k)$  plot shows an **avoided crossing** between the pure photon band (dashed) and the pure TO phonon band (similar to the surface plasmon). An avoided crossing is always a sign that **two bands interact**. At the crossing, a bonding and an antibonding combination of the bands is formed. Their splitting is comparable to the interaction energy.



Since a photon is always transverse, it interacts with the transverse optical (TO) phonon, not with the longitudinal optical (LO) phonon. Indeed, the crossing with the LO band is not avoided.

This plot looks similar to the previous plot, but it is 90° rotated:  $\epsilon(\omega)$  vs.  $\omega(\mathbf{k})$

# Polaritons and the Dielectric Function



The photon resonates with the transverse optical phonon frequency  $\omega_T$  (both transverse).

This plot becomes similar to that for a plasmon (next slide) if one shifts the resonance to  $\omega_T = 0$ .

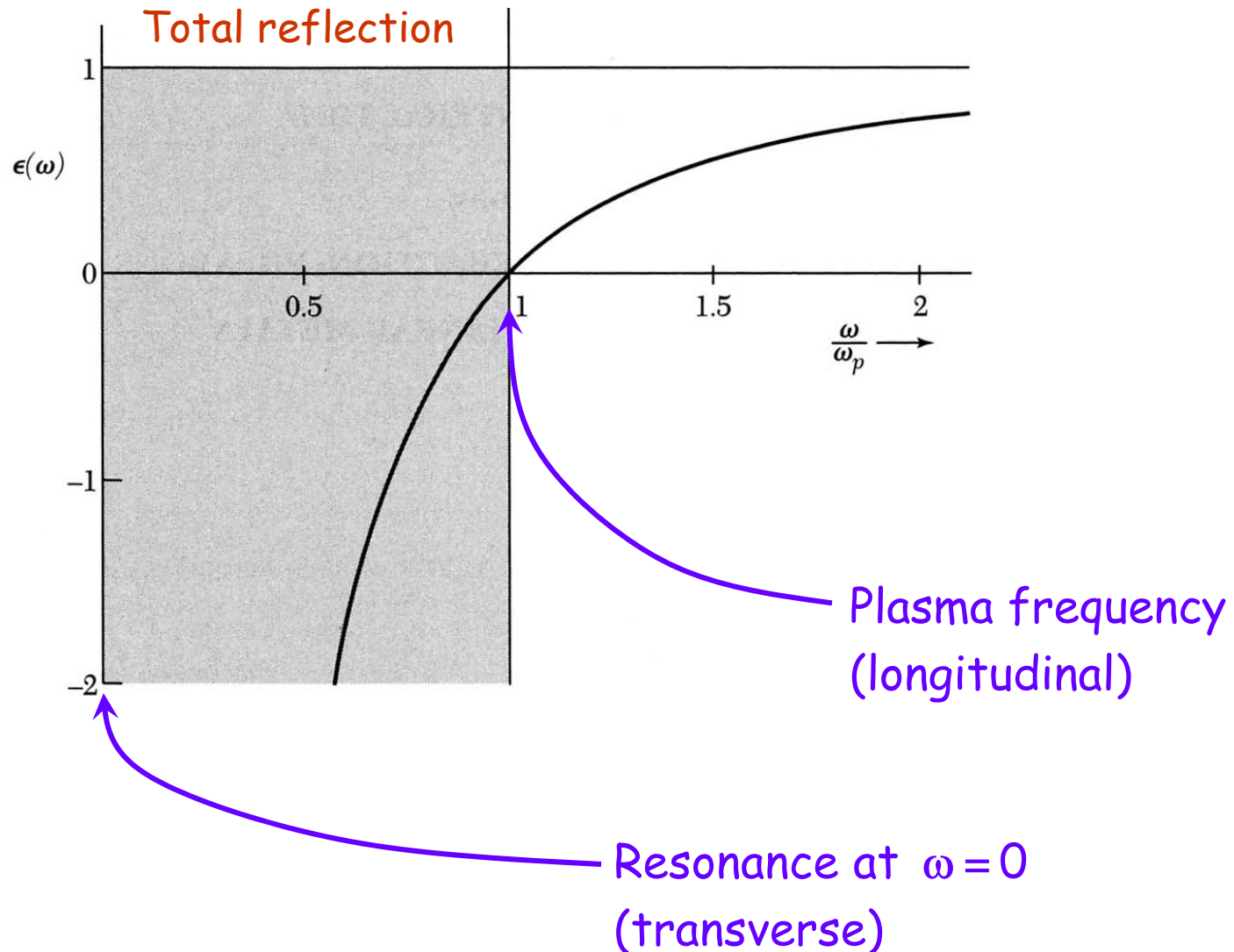
$\omega_L$  is equivalent to  $\omega_p$ . Indeed, both waves are longitudinal.

Longitudinal phonon frequency  
Transverse phonon frequency

**Figure 13a** Plot of  $\epsilon(\omega)$  from (60) for  $\epsilon(\infty) = 2$  and  $\epsilon(0) = 3$ . The dielectric constant is negative between  $\omega = \omega_T$  and  $\omega_L = (3/2)^{1/2}\omega_T$ ; that is, between the pole (infinity) of  $\epsilon(\omega)$  and the zero of  $\epsilon(\omega)$ . Incident electromagnetic waves of frequencies in the shaded regions  $\omega_T < \omega < \omega_L$  will not propagate in the medium, but will be reflected at the boundary.



# A look back at the Dielectric Function of a Metal



# Resonances in the Dielectric Constant $\epsilon$

The dielectric constant is a complex number:  $\epsilon = \epsilon_1 + i \epsilon_2$

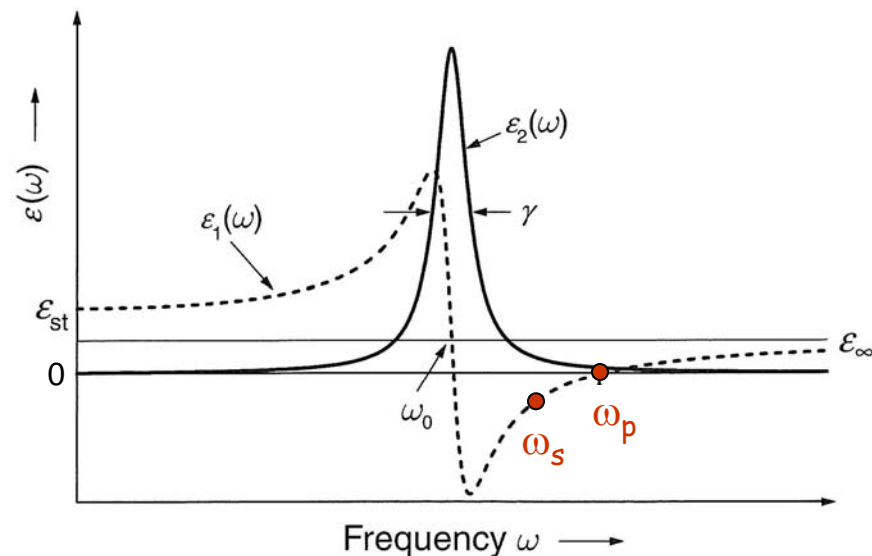
The real part  $\epsilon_1$  describes **refraction** of light (an elastic process)

The imaginary part  $\epsilon_2$  describes **absorption** (inelastic,  $\Delta E \neq 0$ ).

A **bulk plasmon** occurs at  $\omega_p = \omega_L$  where  $\epsilon_1 = 0$ .

A **surface plasmon** occurs at  $\omega_s$  where  $\epsilon_1 = -1$ .

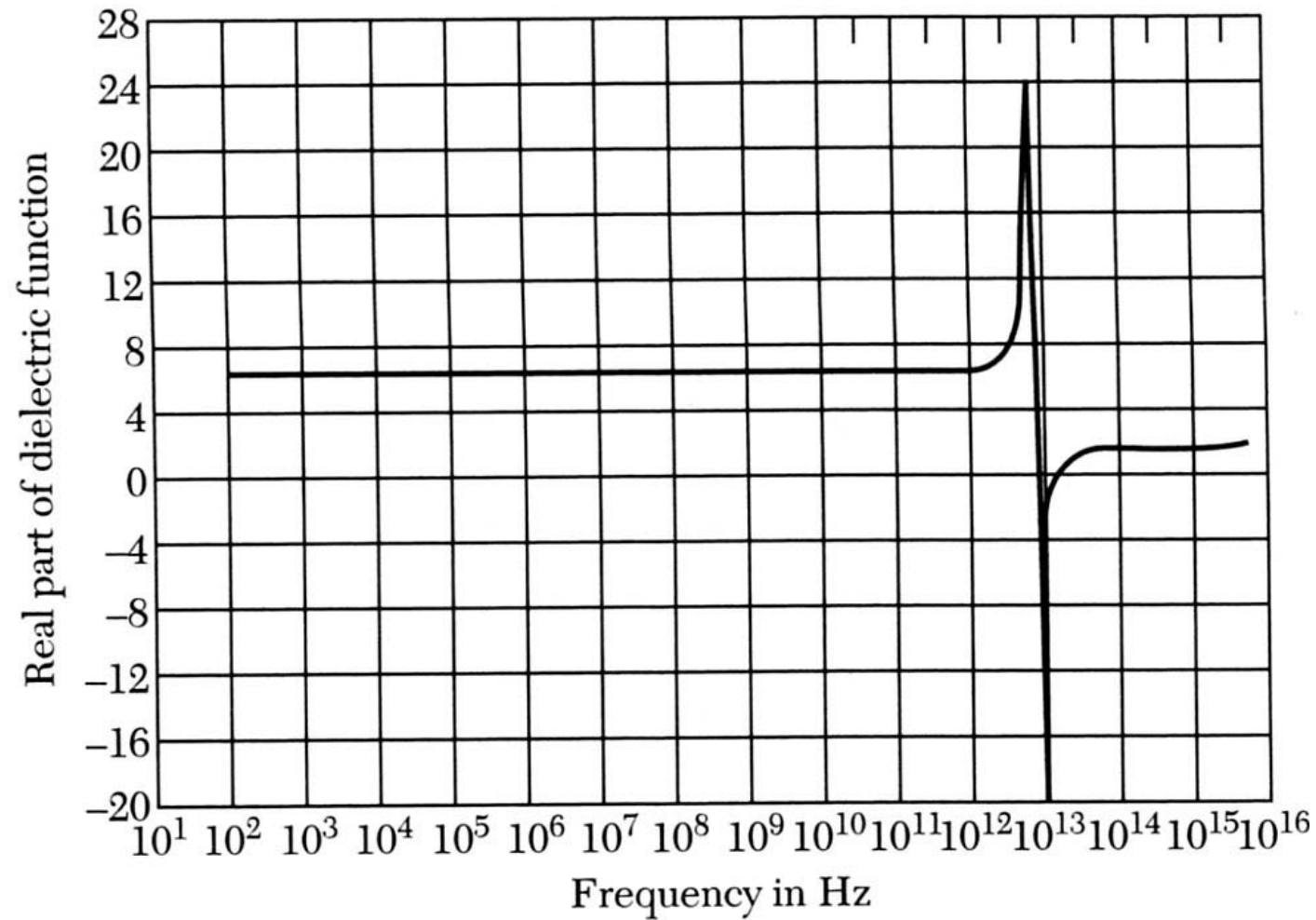
(More precisely:  $\text{Im}[1/\epsilon]$  and  $\text{Im}[1/(\epsilon+1)]$  have maxima.)



General behavior of the dielectric constant  $\epsilon(\omega)$  for a damped oscillator with resonance frequency  $\omega_0$ , static dielectric constant  $\epsilon_{st}$ , asymptotic dielectric constant  $\epsilon_\infty$  ( $\approx n^2$ ,  $n$ =refractive index in the visible), and damping constant  $\gamma > 0$ :

$$\epsilon(\omega) = \epsilon_\infty + \omega_0^2 (\epsilon_{st} - \epsilon_\infty) / (\omega_0^2 - \omega^2 - i\gamma\omega)$$

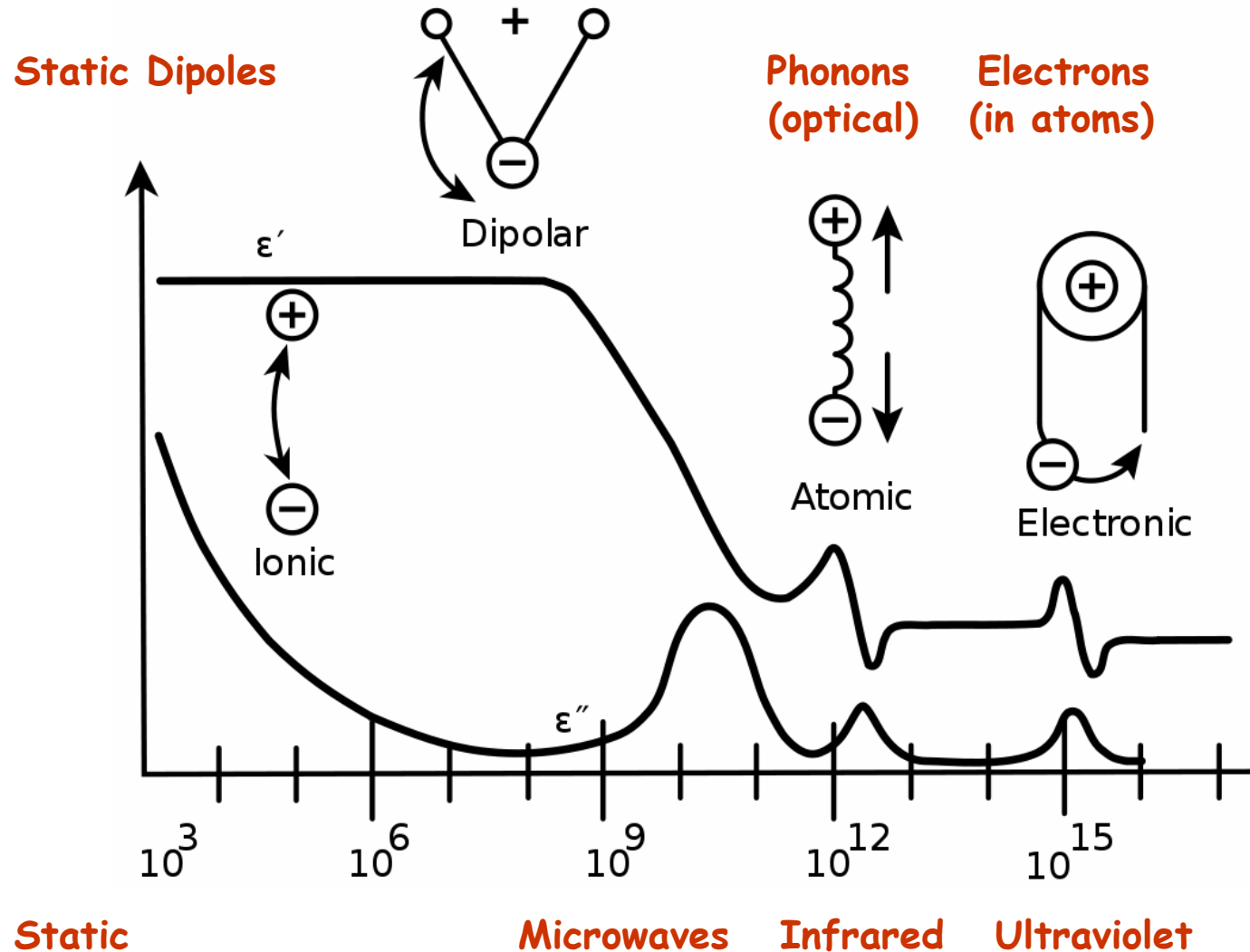
# Polariton Resonance, Experiment



(b)

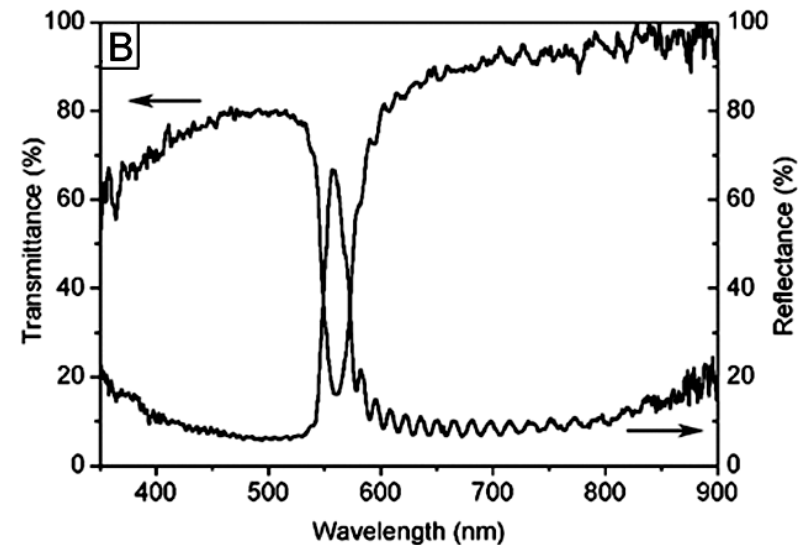
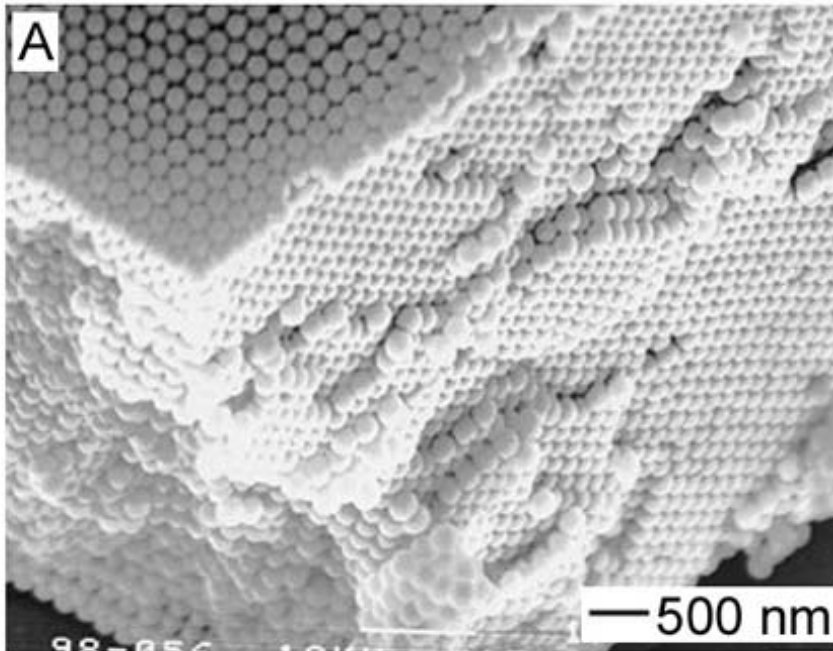
Dielectric function (real part) of  $\text{SrF}_2$

# The Dielectric Constant $\epsilon$ over a Wide Frequency Range



# Photonics

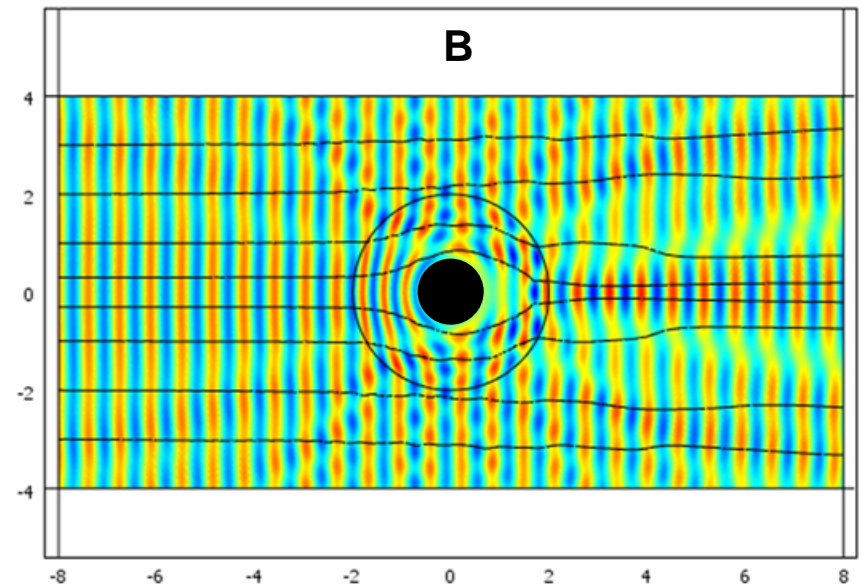
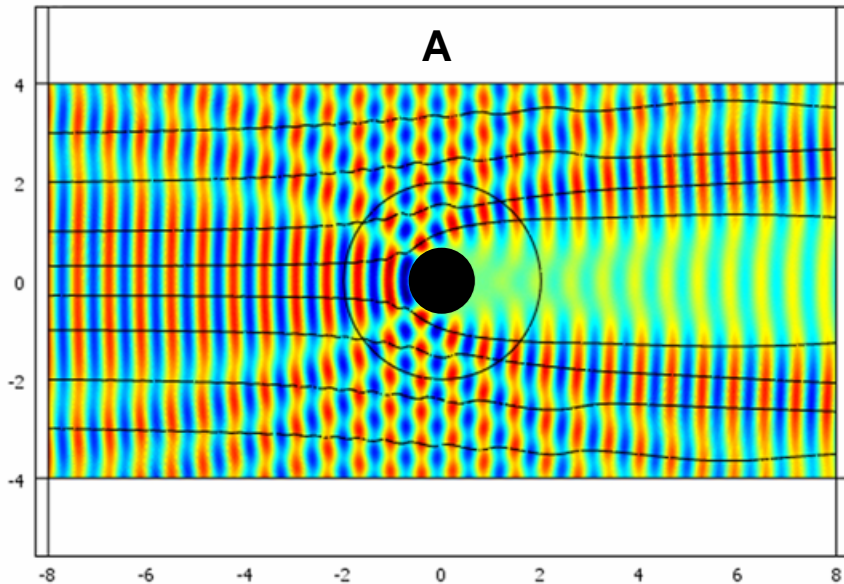
In photonics one tries to manipulate the dielectric constant by creating **artificial resonators** via nano-structured dielectric materials ("metamaterials"). Just above the resonance there is a region with  $\epsilon < 0$  where light is **totally reflected** (Slide 15). This total reflection region can be viewed as a **band gap in the  $E(k)$  relation of polaritons** (Slide 14), analogous to the band gap of electrons in a semiconductor.



An artificial crystal lattice made from polystyrene beads (similar to an opal, an iridescent gemstone). The photonic band gap causes a reflectance maximum.

# Cloaking: Making an Object Invisible

Surrounding an object with a material having the right kind of dielectric properties (negative refractive index) can make the object invisible.



Cloaking simulation in two dimensions:

**A.** The black disc blocks the light coming from the left and reflects it back, leaving a shadow towards the right (shown in light green + yellow).

**B.** The surrounding ring of cloaking material guides the light around the disc. Reflection and shadow are avoided, thereby avoiding any trace of the object.



# Cloaking: Are Harry Potter's Tricks Possible ?

There is still some debate how far one can push the idea of cloaking by materials with a negative refractive index.

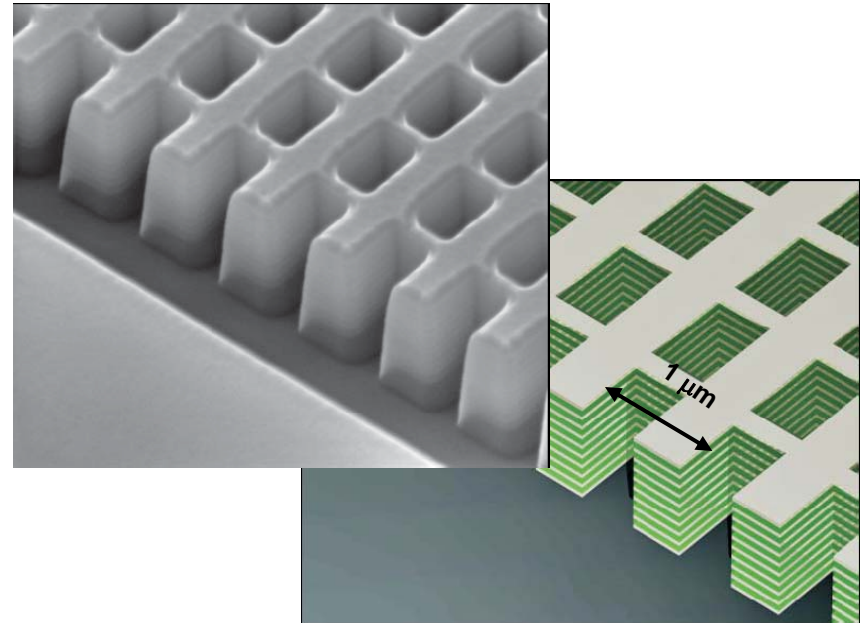
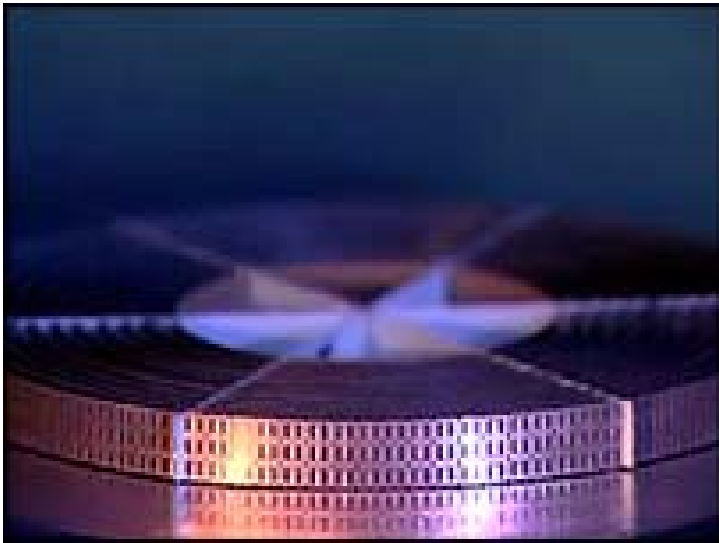
Some observations:

- Cloaking works perfectly only at **one wavelength**, due to the need for a sharp resonance to create negative  $n$ .
- So far, it works mainly with **microwaves**, because the resonators required for visible light would have to be well below a micrometer in size.
- When cloaking just the intensity (but not phase and polarization), the task becomes much simpler.
- If nobody can look through the cloak, can the person wearing it look out?



# Actual Metamaterials

Most metamaterials with negative refractive index have been made for **microwaves** (below left). Such devices are interesting for making an airplane **invisible to radar** (wavelength  $\approx 3$  cm).

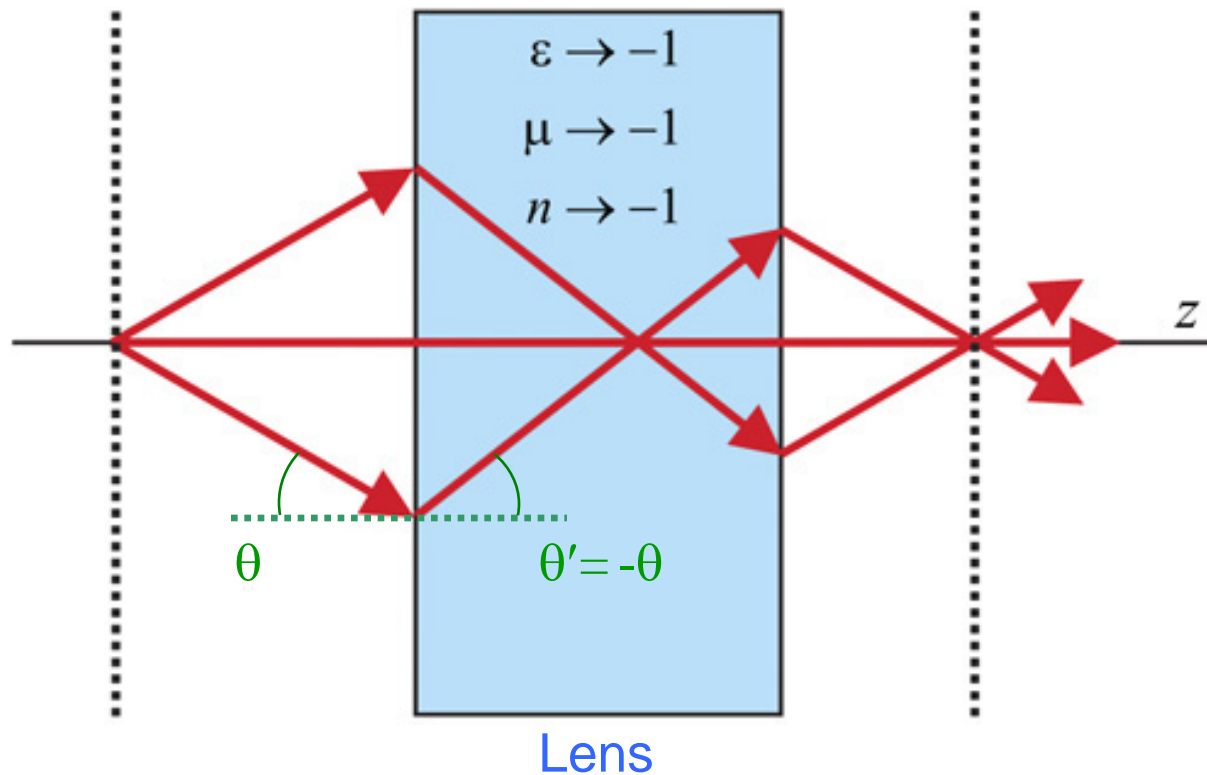


To produce analogous metamaterials for visible light requires nanotechnology with structures small compared the wavelength of light (above right). Even with that under control, it is hard to cloak an object at all wavelengths. Metamaterials are active only near a resonance, which occurs at a particular wavelength.

# The Perfect Lens

A medium with refractive index  $n = -1$  acts as perfect lens.

For  $n = \sqrt{\epsilon} \cdot \sqrt{\mu} = -1$  both  $\epsilon$  and  $\mu$  need to be negative.



Negative  $n$  refracts light towards the same side of the normal (not the opposite side).