Boson Couplings in a Composite Higgs Model

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Abstract

The couplings among bosons are given for a Higgs boson composed of gauge bosons. Compared to the standard model there are three differences: 1) The standard Higgs boson is absent, and with it the self-couplings induced by the standard Higgs potential. 2) The Higgs boson is a composite of the SU(2) gauge bosons. 3) New couplings are introduced for the gauge bosons, because their scalar products acquire finite vacuum expectation values. The absence of a fundamental scalar particle in this model opens new avenues for mitigating problems with the standard Higgs boson, such as radiative corrections that increase quadratically with energy, hierarchy and naturalness.

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1. Background

The discovery of the Higgs boson at CERN in 2012 was a strong boost for the Brout-Englert-Higgs mechanism of symmetry breaking. Since then, there has been enormous experimental and theoretical activity dedicated to detect new physics beyond the standard model. Such efforts aim at expanding our knowledge toward fundamental energy scales, such as the Planck scale of gravity and the unification scale of the three interactions comprising the standard model. These are many orders of magnitude beyond currently accessible energies. It remains unclear how far the standard model can be extrapolated from the narrow observational energy range [1]. Furthermore, the mass of the Higgs boson remains a free parameter of the standard model. Such considerations have led to the development of composite Higgs models [2].

Existing composite Higgs models involve fermions as constituents [2]. In contrast to those, a Higgs boson was defined in [3],[4] that consisted of the three gauge bosons of the weak interaction. In this model, scalar products of gauge boson pairs take finite vacuum expectation values (VEVs) and thereby create their own masses – which then create the mass of the composite Higgs boson. The tree-level mass of the composite Higgs boson becomes half of its VEV v, which is directly related to the four-fermion coupling G_{F}.

The coupling of this composite Higgs boson to fundamental fermions was investigated in [5]. In the Yukawa coupling of the standard model, the Higgs boson was converted to scalar products between gauge bosons and their expectation values. These represented the Higgs – fermion couplings. Likewise, scalar products among gauge boson expectation values produced fermion masses.

In the following, the techniques developed in [3],[4],[5] are employed for converting the standard model to the composite Higgs model. They can be summarized by three basic rules which are to be added to the usual Feynman rules for the standard model:

1) Omit the standard Higgs potential and the resulting Higgs self-interactions.
2) Express the composite Higgs boson in terms of SU(2) gauge bosons.
3) In diagrams containing SU(2) gauge bosons, include their expectation values.

Rule 1 eliminates all vertices originating from the standard Higgs potential. This may remove problems associated with a fundamental Higgs scalar, such as an effective Higgs mass that increases quadratically with the energy scale [6],[7]. The kinetic term of the Higgs remains unchanged. It leads to the propagator of the composite Higgs boson.

Rule 2 converts the standard Higgs couplings to those of the composite Higgs. This is achieved by combining the quadratic relation defining the composite Higgs boson in [3],[4] with a linear relation used in [5] for the Yukawa coupling to fermions.

Rule 3 adds expectation values (EVs) to the gauge bosons. These form vacuum expectation values (VEVs) for scalar products of gauge bosons with themselves, with their EVs, and among their EVs. Those generate masses for fundamental particles.

2. A Brief Review of the Composite Higgs Model

In the standard model, the complex Higgs doublet Φ_0 with the VEV v is given by:

(1) $$\Phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} i w_+ \\ \mathrm{H}_0 - i w_3 \end{pmatrix} \qquad w_+ = w_1 - i w_2 \qquad \mathrm{H}_0 = \langle \mathrm{H}_0 \rangle + \mathrm{H} \qquad \langle \mathrm{H}_0 \rangle = v \qquad \langle w_\mathrm{i} \rangle = 0$$

A pair of composite Higgs bosons was defined by scalar gauge bosons pairs in [3],[4]:

(2) $$\Phi_0^\dagger \Phi_0 \;=\; \tfrac{1}{2}[\mathrm{H}_0^2 + \Sigma_\mathrm{i} w_\mathrm{i}^2] \;=\; -g^2 \cdot \tfrac{1}{2} \Sigma_\mathrm{i} (\mathrm{W}_0^\mathrm{i} \mathrm{W}_0^\mathrm{i})$$

The subscript 0 indicates that these are the lagrangian bosons, which are massless but exhibit a finite expectation values (EVs). After subtracting their EVs one obtains the observable bosons, which are massive and lack an EV (shown without the subscript 0). Gauge invariance requires a specific form of the SU(2) gauge bosons W_0^i and their EVs, consisting of scalar fields w^i multiplied by a common transverse polarization vector ε_α:

(3) $$\mathrm{W}_0^\mathrm{i} = \langle \mathrm{W}_0^\mathrm{i} \rangle + \mathrm{W}^\mathrm{i} \qquad \langle \mathrm{W}_0^\mathrm{i} \rangle = w \cdot \varepsilon_\alpha \qquad \mathrm{W}^\mathrm{i} = \mathrm{w}^\mathrm{i} \cdot \varepsilon_\alpha \qquad (\varepsilon_\alpha^* \varepsilon_\beta) = -\delta_{\alpha\beta} \qquad (\mathrm{W}^\mathrm{i} \mathrm{W}^\mathrm{i}) = -(\mathrm{w}^\mathrm{i})^2$$

By moving the Goldstones w_i to the right side of Eq. (2) via the unitary gauge, the gauge bosons W^i acquire longitudinal components $\mathrm{W}_\mathrm{L}^\mathrm{i}$. Like the w_i, they have vanishing EVs.

After decomposing H_0 and the W_0^i into VEVs and observable bosons, one can expand each side of (2) into a Taylor series (with H/v and w^i/w as small quantities). The three leading terms provide three separate constraints [3],[4]. These have the form VEV·VEV, VEV·Boson, and Boson·Boson:

(4) $$v^2 \;\approx\; -g^2 \cdot \Sigma_\mathrm{i} (\langle \mathrm{W}_0^\mathrm{i} \rangle \langle \mathrm{W}_0^\mathrm{i} \rangle) \qquad \boxed{v \approx \sqrt{3}\, g w}$$

(5) $vH \approx -g^2\cdot\Sigma_i(\langle W_0^i\rangle W^i)$ $\boxed{H \approx (g/\sqrt{3})\cdot\Sigma_i w^i}$

(6) $\boxed{[H^2+\Sigma_i w_i^2] \approx -g^2\cdot\Sigma_i(W^iW^i)}$

3. Interactions among the Composite Higgs Boson and the Gauge Bosons

First, one has to eliminate the standard Higgs potential from the Lagrangian. It gets replaced by the gauge boson potential introduced in [3]. The remaining couplings between the composite Higgs boson and the gauge bosons originate from the gauge-invariant derivative D_μ in the kinetic Higgs Lagrangian:

(7) $L_{\Phi 0} = (D_\mu\Phi_0)^\dagger\cdot(D^\mu\Phi_0)$ $D_\mu\Phi_0 = [\partial_\mu + ig\cdot\Sigma_j \tfrac{1}{2}\tau^j\cdot W^j_{0,\mu}]\,\Phi_0$

D_μ contains the weak coupling g and the Pauli matrices τ^j, which connect the gauge boson triplet with the Higgs doublet.

(8) $L_{\Phi 0} = (\partial_\mu\Phi_0)^\dagger\cdot(\partial^\mu\Phi_0) + \tfrac{1}{4}g^2\cdot(\Phi_0^\dagger\Phi_0)\cdot(\Sigma_j W^j_{0,\mu}W_0^{j,\mu})$

Gauge fixing terms and ghost fields [6] have been omitted for clarity. Focusing on the observable Higgs boson H and its VEV v, the decomposition $H_0=(v+H)$ in (1) produces the kinetic term of H, the quadruple vertex HHW_0W_0, the triple vertex HW_0W_0, and the quadratic term W_0W_0, which contains the mass terms of the observable gauge bosons W^j:

(9) $L_H = \tfrac{1}{2}(\partial_\mu H)(\partial^\mu H) +$

$$+ \tfrac{1}{8}g^2\cdot(HH)\cdot(\Sigma_j W^j_{0,\mu}W_0^{j,\mu}) + \tfrac{1}{4}g^2\cdot(vH)\cdot(\Sigma_j W^j_{0,\mu}W_0^{j,\mu}) + \tfrac{1}{8}g^2\cdot v^2\cdot(\Sigma_j W^j_{0,\mu}W_0^{j,\mu})$$

The terms proportional to $\mu^2H_0^2$ and $-\lambda H_0^4$ from the standard Higgs potential are absent.

The additional decomposition $W_0^i=(w^i+w)\cdot\varepsilon_\alpha$ of the gauge bosons in (3) yields the Lagrangian for the composite Higgs boson and its interactions with the gauge bosons:

(10) $L_H = \tfrac{1}{2}(\partial_\mu H)(\partial^\mu H) +$

$$+ \tfrac{1}{8}g^2\cdot(HH)\cdot(\Sigma_j W^j_\mu W^{j,\mu}) + \tfrac{1}{4}g^2\cdot(vH)\cdot(\Sigma_j W^j_\mu W^{j,\mu}) + \tfrac{1}{8}g^2\cdot v^2\cdot(\Sigma_j W^j_\mu W^{j,\mu})$$
$$- \tfrac{1}{4}g^2\cdot(HH)\cdot(w\,\Sigma_j w^j) \quad - \tfrac{1}{2}g^2\cdot(vH)\cdot(w\,\Sigma_j w^j) \quad - \tfrac{1}{4}g^2\cdot v^2\cdot(w\,\Sigma_j w^j)$$
$$- \tfrac{1}{8}g^2\cdot(HH)\cdot 3w^2 \quad - \tfrac{1}{4}g^2\cdot(vH)\cdot 3w^2 \quad - \tfrac{1}{8}g^2\cdot v^2\cdot 3w^2$$

The last two lines are generated by the VEV w of the gauge bosons. The minus signs reflect the space-like polarization vector of the gauge bosons ($\varepsilon_\alpha^*\varepsilon_\alpha=-1$).

At this point one could consider two additional conversions. Either eliminate the composite Higgs boson H in (10) by replacing it with gauge bosons using (4),(5):

(11) $\quad v \to (\sqrt{3}g)\cdot w \qquad \mathrm{H} \to (g/\sqrt{3})\cdot \Sigma_j \mathrm{w}^j$

Or make the inverse substitution via (4),(5),(6) to eliminate the gauge bosons:

(12) $\quad w \to v/(\sqrt{3}g) \qquad \Sigma_j \mathrm{w}^j \to (\sqrt{3}/g)\cdot \mathrm{H} \qquad (\Sigma_j \mathrm{W}^j_\mu \mathrm{W}^{j,\mu}) \to -(\mathrm{HH})/g^2$

The latter yields an effective potential for the composite Higgs boson:

(13)
$$
\begin{aligned}
L_\mathrm{H} = \tfrac{1}{2}(\partial_\mu \mathrm{H})(\partial^\mu \mathrm{H}) &- \tfrac{1}{8}\cdot(\mathrm{HH})\cdot(\mathrm{HH}) - \tfrac{1}{4}\cdot(v\mathrm{H})\cdot(\mathrm{HH}) - \tfrac{1}{8}\cdot v^2\cdot(\mathrm{HH}) \\
&- \tfrac{1}{4}\cdot(\mathrm{HH})\cdot(v\mathrm{H}) - \tfrac{1}{2}\cdot(v\mathrm{H})\cdot(v\mathrm{H}) - \tfrac{1}{4}\cdot v^2\cdot(v\mathrm{H}) \\
&- \tfrac{1}{8}\cdot(\mathrm{HH})\cdot v^2 \quad - \tfrac{1}{4}\cdot(v\mathrm{H})\cdot v^2 \quad - \tfrac{1}{8}\cdot v^2\cdot v^2 \\
= \tfrac{1}{2}(\partial_\mu \mathrm{H})(\partial^\mu \mathrm{H}) &- \tfrac{1}{8}\mathrm{H}^4 - \tfrac{1}{2}v\mathrm{H}^3 - \tfrac{3}{4}v^2\mathrm{H}^2 - \tfrac{1}{2}v^3\mathrm{H} - \tfrac{1}{8}v^4 \\
= \tfrac{1}{2}(\partial_\mu \mathrm{H})(\partial^\mu \mathrm{H}) &- \tfrac{1}{8}(\mathrm{H}+v)^4 \quad = \quad \tfrac{1}{2}(\partial_\mu \mathrm{H}_0)(\partial^\mu \mathrm{H}_0) - \tfrac{1}{8}\mathrm{H}_0^4 \quad = \quad L_{\mathrm{H}0}
\end{aligned}
$$

In contrast to the standard model, the quadratic term is absent in $L_{\mathrm{H}0}$. Only a higher order H_0^4 term remains.

4. Conclusion

In summary, this work is meant to facilitate calculations involving the composite Higgs model. Three simple rules are provided for the overall changes to the standard model. Specific information is given here about the boson-boson couplings in this model. Combined with the Lagrangians for the Higgs-fermion couplings from [5], this enables minimal modifications to codes developed for the standard model.

To get started, one could simply omit the composite Higgs boson from the diagrams for the gauge boson potential given in [3]. That makes sense, because the definition (2) shows that the amplitude of the composite Higgs boson is down by a factor of g from that of the gauge bosons. It is also consistent with the poor sensitivity of precision measurements to the Higgs mass, which has made the search for the Higgs boson more difficult. Already the signs of the quadratic and quartic terms of the gauge boson potential would provide an important self-consistency check. As pointed out in [3], early calculations of the quadratic gauge boson self-energy had already the correct sign.

The extension from SU(2) to the SU(2)×U(1) symmetry of the electroweak interaction follows standard procedures, as outlined in [3],[6]. A further extension to the complete SU(3)×SU(2)×U(1) standard model can be accomplished by using the Higgs-fermion coupling to fermions in [5] for the quarks.

References

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