Boson Couplings in a Composite Higgs Model

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Abstract
The couplings among bosons are given for a Higgs boson composed of gauge bosons. Compared to the standard model there are three differences: 1) The standard Higgs boson is absent, and with it the self-couplings induced by the standard Higgs potential. 2) The Higgs boson is a composite of the SU(2) gauge bosons. 3) New couplings are introduced for the gauge bosons, because their scalar products acquire finite vacuum expectation values. The absence of a fundamental scalar particle in this model opens new avenues for mitigating problems with the standard Higgs boson, such as radiative corrections that increase quadratically with energy, hierarchy and naturalness.
1. Background

The discovery of the Higgs boson at CERN in 2012 was a strong boost for the Brout-Englert-Higgs mechanism of symmetry breaking. Since then, there has been enormous experimental and theoretical activity dedicated to detect new physics beyond the standard model of particle physics. An overarching goal of such efforts aims at expanding our knowledge toward fundamental energy scales, such as the Planck scale of gravity or the unification scale of the three interactions comprising the standard model. These are many orders of magnitude beyond currently accessible energies. It remains unclear how far the standard model can be extrapolated from the narrow observational energy range [1]. Also, the observed mass of the Higgs boson remains a free parameter of the standard model. Such considerations have led to the development of composite Higgs models [2].

While most composite Higgs models involve fermions as constituents [2], a Higgs boson composed of the three gauge bosons $W^i$ of the weak interaction was defined in [3]. In this model, scalar products of $W^i$ pairs create their own masses by taking on finite vacuum expectation values (VEVs) – analogous to the Brout-Englert-Higgs mechanism for the Higgs boson. As a result, the tree-level mass boson becomes half of its VEV $v$, which is directly related to the four-fermion coupling $G_F$. The coupling of this composite Higgs boson to fundamental fermions was investigated in [4]. In the Yukawa coupling of the standard model, the Higgs boson was converted to a sum of scalar products between the vector bosons $W^i$ and their polarization vectors.

In the following, the techniques developed in [3],[4] are employed for converting the standard model to the composite Higgs model. They can be summarized by three basic rules which are to be added to the usual Feynman rules for the standard model:

1) Omit all diagrams containing the standard Higgs boson.
2) Express the composite Higgs boson in terms of SU(2) gauge bosons.
3) In diagrams containing SU(2) gauge bosons, include their expectation values.

Rule 1 eliminates all vertices originating from the standard Higgs potential. This removes problems associated with a fundamental Higgs scalar, such as an effective Higgs mass that increases quadratically with the energy scale [5],[6].
Rule 2 converts the standard Higgs couplings to those of the composite Higgs. This is achieved by combining the quadratic relation defining the composite Higgs boson in [3] with a linear relation used in [4] for the Yukawa coupling to fermions.

Rule 3 adds expectation values (EVs) to the gauge bosons. These form vacuum expectation values (VEVs) for scalar products of gauge bosons with themselves and with their EVs. These scalar VEVs generate masses for fundamental particles via an extension of the Brout-Englert-Higgs mechanism, as shown in [3],[4].

2. A Brief Review of the Composite Higgs Model

In the standard model, the complex Higgs doublet $\Phi_0$ with the VEV $v$ is given by:

$$\Phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} i w_+ \\ H_0 - i w_3 \end{pmatrix} \quad w_+ = w_1 - i w_2 \quad H_0 = \langle H_0 \rangle + H \quad \langle H_0 \rangle = v \quad \langle w_i \rangle = 0$$

The composite Higgs boson is defined by scalar products of the gauge bosons $W_0^i$ [2],[3]:

$$\Phi_0^\dagger \Phi_0 = \frac{1}{2} [H_0^2 + \Sigma_i w_i^2] = -g^2 \cdot \frac{1}{2} \Sigma_i (W_0^i W_0^i)$$

The subscript 0 indicates that these are the lagrangian bosons, which are massless but exhibit a finite expectation values (EVs). After subtracting their EVs one obtains the observable bosons, which are massive and lack an EV (shown without the subscript). Gauge invariance requires a specific form of the SU(2) gauge bosons $W_0^i$ and their EVs, consisting of scalar fields $w^i$ multiplied by a common transverse polarization vector $\epsilon_\alpha$:

$$W_0^i = \langle W_0^i \rangle + W_1^i \quad \langle W_1^i \rangle = w \cdot \epsilon_\alpha \quad W_1^i = w^i \cdot \epsilon_\alpha$$

$$\langle \epsilon_\alpha^* e_\beta \rangle = -\delta_{\alpha \beta} \quad (W_1^i W_1^i) = -(w^i)^2$$

By moving part of the Goldstone term $\Sigma_i w_i^2$ in (2) to the right, the observable gauge bosons $W_i^i$ may acquire a longitudinal component $W_L^i$ in certain gauges.

After decomposing $H_0$ and the $W_0^i$ into VEVs and observable bosons, one can expand each side of (2) into a Taylor series (with $H/v$ and $W_i/w$ as small quantities). The three leading terms provide three separate constraints [3],[4]. These have the form VEV-VEV, VEV-Boson, and Boson-Boson:

$$v^2 \approx -g^2 \cdot \Sigma_i (\langle W_0^i \rangle \langle W_0^i \rangle)$$

$$v H \approx -g^2 \cdot \Sigma_i (\langle W_0^i \rangle W_1^i)$$

$$[H^2 + \Sigma_i w_i^2] \approx -g^2 \cdot \Sigma_i (W_i W_i)$$
3. Interactions among the Composite Higgs Boson and the Gauge Bosons

In the following we will use the relations (4),(5),(6) to eliminate the standard Higgs H from all of its interactions with gauge bosons. The couplings among the Higgs and gauge bosons originate from the gauge-invariant derivative $D_{\mu}$ in the Lagrangian of the composite Higgs boson:

$$L_{\Phi_0} = (D_{\mu} \Phi_0)^\dagger (D^\mu \Phi_0)$$

$D_{\mu}$ contains the weak coupling $g$ and the Pauli matrices $\tau^i$ which connect the gauge boson triplet with the Higgs doublet. In the unitary gauge the Goldstones $w_j$ have been converted into longitudinal gauge bosons $W^j_L$. $L_{\Phi_0}$ then depends only on $H_0$ and $W_0^j$:

$$L_{H_0} = \frac{1}{2} (\partial_{\mu} H_0)(\partial^{\mu} H_0) + \frac{1}{6} g^2 (H_0 H_0)(\Sigma_j W_0^j W_0^{j\mu}) + \frac{1}{8} g^2 (H_0 H_0)(\Sigma_j W_0^j L W_0^{j\mu})$$

The standard model includes only the VEV $v$ of the Higgs boson via the decomposition $H_0 = v + H$, with $W_0^j = W_T^j$ and $(W_T^j + W_L^j) = W_j$. That yields the kinetic term of the $H$, the quadruple vertex $HHWW$, the triple vertex $HWW$, and the mass terms of the $W_j$:

$$L_H = \frac{1}{2} (\partial_{\mu} H)(\partial^{\mu} H) +$$

$$+ \frac{1}{6} g^2 (HH)(\Sigma_j W_{ij} W^{ij\mu}) + \frac{1}{4} g^2 (v H)(\Sigma_j W_{ij} W^{ij}\mu) + \frac{1}{6} g^2 (v^2)(\Sigma_j W_{ij} W^{ij\mu})$$

In the composite Higgs model the gauge bosons acquire the EV $w_\alpha$ according to (3).

With the additional decomposition $W_0^j = (w + w^j) \cdot e_\alpha$ from (3) one obtains:

$$L_H = \frac{1}{2} (\partial_{\mu} H)(\partial^{\mu} H) +$$

$$+ \frac{1}{6} g^2 (HH)(\Sigma_j W_{ij} W^{ij\mu}) + \frac{1}{4} g^2 (v H)(\Sigma_j W_{ij} W^{ij\mu}) + \frac{1}{6} g^2 (v^2)(\Sigma_j W_{ij} W^{ij\mu})$$

$$- \frac{1}{4} g^2 (HH)(w \Sigma_j W^j) - \frac{1}{2} g^2 (v H)(w \Sigma_j W^j) - \frac{1}{4} g^2 (v^2)(w \Sigma_j W^j)$$

$$- \frac{1}{8} g^2 (HH) \cdot 3 w^2 - \frac{1}{4} g^2 (v H) \cdot 3 w^2 - \frac{1}{8} g^2 \cdot v^2 \cdot 3 w^2$$

The last two lines are generated by the VEV $w$ of the gauge bosons. The minus signs reflect the space-like polarization vector of the gauge bosons ($e_\alpha^* e_\alpha = -1$). These lines contain exclusively scalars. At this point there are two options. One can eliminate the composite Higgs boson completely by replacing it with gauge bosons using (4),(5):

$$v \rightarrow (\sqrt{3} g) \cdot w \quad H \rightarrow (g/\sqrt{3}) \cdot \Sigma_j w^j$$

Making the inverse substitution via (4),(5),(6) yields a pure Higgs Lagrangian (see [7]):

$$w \rightarrow v/(\sqrt{3} g) \quad \Sigma_j w^j \rightarrow (\sqrt{3}/g) \cdot H \quad (\Sigma_j W_{ij} W^{ij\mu}) \rightarrow -(HH)/g^2$$

Either way, (10) describes the bosonic couplings of the composite Higgs boson.
4. Conclusion

In summary, this work is meant to facilitate calculations involving the composite Higgs model. Three simple rules are provided for the overall changes to the standard model. Specific information is given about the new boson-boson couplings. Combined with the Lagrangians for the Higgs-fermion vertices from [4], this enables minimal modifications to codes developed for the standard model. It will be of particular interest to have calculations of the quadratic and quartic SU(2) gauge boson self-interactions. Those provide a theoretical test of the composite Higgs model, as elaborated in [3]. The extension from SU(2) to the SU(2)×U(1) symmetry of the electroweak interaction has been outlined there as well. A further extension to the complete SU(3)×SU(2)×U(1) symmetry of the standard model involves only standard vertices.

References


7. The Lagrangian for the composite Higgs boson resulting from (10),(12) becomes:

\[ L_H = \frac{1}{2} (\partial_\mu H)(\partial^\mu H) - \frac{i}{6} \cdot (HH) \cdot (HH) - \frac{i}{4} \cdot (vH) \cdot (HH) - \frac{i}{6} \cdot v^2 \cdot (HH) - \frac{1}{4} \cdot (HH) \cdot v^2 \cdot (HH) - \frac{1}{4} \cdot (vH) \cdot v^2 \cdot (HH) - \frac{1}{4} \cdot v^2 \cdot v^2 \]

\[ = \frac{1}{2} (\partial_\mu H)(\partial^\mu H) - \frac{i}{6} H^4 - \frac{i}{2} v H^3 - \frac{1}{4} v^2 H^2 - \frac{1}{2} v^3 H - \frac{i}{6} v^4 \]

\[ = \frac{1}{2} (\partial_\mu H)(\partial^\mu H) - \frac{i}{6} (H+v)^4 = \frac{1}{2} (\partial_\mu H_0)(\partial^\mu H_0) - \frac{i}{6} H_0^4 = L_{H0} \]

Notice that the quadratic term of the standard Higgs potential is absent.