Fermion Masses and Couplings in a Composite Higgs Model

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Abstract

This work explores the coupling of fundamental fermions (quarks and leptons) to a previously-proposed Higgs boson, which is composed of SU(2) gauge bosons. In the standard model, the fermion masses are generated via coupling to the vacuum expectation value of the Higgs boson. It is shown that the gauge bosons forming the composite Higgs boson are also able to perform this task. The definition of the composite Higgs ensures that the resulting fermion masses remain unchanged from those of the standard model. With the inclusion of fundamental fermions, the composite Higgs model covers the complete particle range of the standard model.

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1. Background

A composite Higgs boson was defined in Refs. [1],[2] by identifying a pair of Higgs bosons with a superposition of SU(2) gauge bosons pairs. The Higgs potential of the standard model was replaced by quadratic and quartic one-loop self-interactions of the SU(2) gauge bosons. This was accomplished by a generalization of the Brout-Englert-Higgs mechanism from scalars to vectors. In order to preserve Lorentz invariance of the vacuum, the expectation value (EV) of each individual gauge boson was assumed to be proportional to its polarization vector. The summation over the polarizations of all virtual vacuum bosons then led to a vanishing vacuum expectation value (VEV) for vectors. Nevertheless, scalar pairs of vector bosons exhibited a common VEV which did not average out to zero. That allowed dynamical symmetry breaking analogous to the standard Brout-Englert-Higgs mechanism.

Since gauge bosons create their own masses in this model (like the Higgs boson), one may ask whether they are also able to generate masses for the fundamental fermions (quarks and leptons). The standard Higgs boson produces masses for fundamental fermions from the Yukawa coupling of its VEV to fermions. It turns out that the scalar product of a gauge boson with its EV yields a finite scalar boson that is proportional to the composite Higgs boson. This relation can be used to replace the standard Higgs boson by its composite version in the Yukawa coupling. The fermion masses are then determined by the gauge boson VEV. The results are the same as in the standard model when the relation between the VEVs of the gauge bosons and the composite Higgs boson is used. Thus, the concept of a Higgs boson composed of SU(2) gauge bosons can be generalized from fundamental bosons to fundamental fermions.

2. Fermion Mass Terms and Couplings

To simplify the discussion we take the top/bottom quark doublet q=(t,b) as example for all other fermion SU(2) doublets. These are the heaviest fermions, which exhibit the strongest couplings to the Higgs boson in the standard model. Their mass Lagrangian has the form (with round brackets designating a scalar product):

(1) $L_m = -m_\mathrm{t}\cdot(\bar{\mathrm{t}}\mathrm{t}) - m_\mathrm{b}\cdot(\bar{\mathrm{b}}\mathrm{b})$

In the standard model [4], the mass m_f of a fundamental fermion is proportional to its Yukawa coupling g_f to the complex Higgs doublet Φ_0 (with the VEV v):

(2) $g_f = \sqrt{2}\, m_f / v$

$$\Phi_0 = \frac{1}{\sqrt{2}} \begin{bmatrix} i\,w_+ \\ H_0 - i\,w_3 \end{bmatrix} \qquad w_+ = w_1 - i\,w_2 \qquad H_0 = \langle H_0\rangle + H \qquad \langle H_0\rangle = v \qquad \langle w_i\rangle = 0$$

The Lagrangian $L_{\Phi,q}$ of the Yukawa coupling has the following form:

(3a) $$L_{\Phi,q} = -g_t \cdot [(\overline{q}_L \Phi_0^C)\, t_R + \overline{t}_R (\Phi_0^{C\dagger} q_L)] \qquad g_t = \sqrt{2}\, m_t/v \qquad g_b = \sqrt{2}\, m_b/v$$

$$-g_b \cdot [(\overline{q}_L \Phi_0)\, b_R + \overline{b}_R (\Phi_0^{\dagger} q_L)] \qquad q_L = ½(1-\gamma^5)\cdot q \qquad q_R = ½(1+\gamma^5)\cdot q$$

In the unitary gauge the Goldstones w_i vanish, and the Yukawa coupling simplifies to:

(3b) $$L_{H,q} = -\frac{1}{\sqrt{2}} \{ g_t \cdot [\, \overline{t}_L H_0 t_R + \overline{t}_R H_0 t_L] + g_b \cdot [\, \overline{b}_L H_0 b_R + \overline{b}_R H_0 b_L] \}$$

$$= -\frac{1}{\sqrt{2}} \,[g_t \cdot (\,\overline{t} H_0 t) + g_b \cdot (\,\overline{b} H_0 b)] \qquad \text{Unitary gauge}$$

The substitution $H_0 = (v + H)$ produces two terms:

(4) $$L_{H,q} = -[\; m_t \cdot (\,\overline{t}t) \;+\; m_b \cdot (\,\overline{b}b) \;] \qquad m_t = g_t \cdot v/\sqrt{2} \qquad m_b = g_b \cdot v/\sqrt{2}$$

$$-\frac{1}{\sqrt{2}} [g_t \cdot (\,\overline{t} H t) + g_b \cdot (\,\overline{b} H b)]$$

The first line corresponds to the fermion mass Lagrangian, and the second line to the fermion-Higgs coupling. The experimental values of m_t and v from [3] yield $g_t = 0.9925$, which is consistent with unity. From the analogous relation for the bottom quark one obtains a much smaller bottom-Higgs coupling $g_b = 0.0240$. The ratio is comparable to the typical tree-level accuracy $\alpha_w = g^2/4\pi \approx 3\%$, i.e., the mass of the bottom quark may be viewed as zero in that approximation.

The composite Higgs boson is defined via scalar products of gauge bosons [2]:

(5) $$\Phi_0^{\dagger}\Phi_0 \;=\; ½\,[H_0^2 + \Sigma_i w_i^2] \;=\; -g^2 \cdot ½\, \Sigma_i (W_0^i W_0^i) \qquad \text{General gauge}$$

The three SU(2) gauge bosons W_0^i are decomposed into their expectation values $\langle W_0^i\rangle$ and the transverse parts W_T^i of the observable gauge bosons W^i. Being transverse preserves gauge invariance [1],[2]. They are written as products of scalar fields w^i and a common polarization vector ε_α (α=1,2):

(6) $$W_0^i = \langle W_0^i\rangle + W^i \qquad \langle W_0^i\rangle = w \cdot \varepsilon_\alpha \qquad W^i = w^i \cdot \varepsilon_\alpha \qquad (\varepsilon_\alpha^* \varepsilon_\beta) = -\delta_{\alpha\beta} \qquad (W^i W^i) = -(w^i)^2$$

The Goldstones w_i in (5) can be converted to longitudinal gauge bosons W_L^i, depending on the gauge. Both have vanishing expectation values and do not affect fermion masses.

The bosonic Feynman diagrams are obtained by applying three basic rules [2]:

1) Omit all diagrams containing the standard Higgs boson.
2) Define the composite Higgs boson H_0 in terms of the SU(2) gauge bosons W_0^i via (5).
3) Add the expectation values $\langle W_0^i \rangle$ to the observable gauge bosons W^i via (6).

For fermionic diagrams one has to apply the 2nd rule to the Yukawa couplings in (3),(4). This is accomplished by expanding both sides of (5) into a Taylor series, using H/v and W^i/w as small quantities. The three leading terms of the expansion have the form VEV·VEV, VEV·Boson, and Boson·Boson [2]:

(7) $v^2 \approx -g^2 \cdot \Sigma_i (\langle W_0^i \rangle \langle W_0^i \rangle)$ $\quad \boxed{v \approx \sqrt{3}\, g w}$ General gauge

(8) $v H \approx -g^2 \cdot \Sigma_i (\langle W_0^i \rangle W_T^i)$ $\quad \boxed{H \approx (g/\sqrt{3}) \cdot \Sigma_i w^i}$ General gauge

(9) $[H^2 + \Sigma_i w_i^2] \approx -g^2 \cdot \Sigma_i [(W_T^i W_T^i) + (W_L^i W_L^i)]$ General gauge

To determine fermion masses in the composite Higgs model, one simply uses (7) for converting the Higgs VEV v to the gauge boson VEV w in (4). Likewise, the Higgs boson H in the fermion couplings is converted to the scalar bosons w^i using (7),(8):

(10) $L_{W,q} = -[\, m_t \cdot (\bar{t} t) + m_b \cdot (\bar{b} b)\,]$ $\quad m_t = g_t \cdot \sqrt{3}\, g w / \sqrt{2}$ $\quad m_b = g_b \cdot \sqrt{3}\, g w / \sqrt{2}$

$$- \frac{1}{\sqrt{2}} [\, g_t \cdot (g/\sqrt{3}) \cdot \Sigma_i (\bar{t} w^i t) + g_b \cdot (g/\sqrt{3}) \cdot \Sigma_i (\bar{b} w^i b)\,]$$

As a result, fermions couple to the scalar part w^i of the gauge bosons in (6), rather than to the standard Higgs boson. Nevertheless, the relations (7),(8) ensure that the resulting fermion masses and couplings remain the same as in the standard model.

3. Conclusion

This essay completes the general framework for replacing the Higgs boson of the standard model by a composite of intrinsic SU(2) gauge bosons. Its interactions with both bosons and fermions are now established, and the connection to the standard model has been made. Of course, there are many important technical details to be worked out. The most pressing tasks are calculations of the quadratic and quartic self-interactions of the SU(2) gauge bosons that determine the spontaneous breakdown of the SU(2) gauge symmetry. The results could lead to a convincing theoretical argument deciding the validity of the concept. An experimental confirmation would probably require detailed data near the 250 GeV threshold for creating Higgs pairs. This is also close to the VEV of the Higgs boson. While further upgrades of the LHC might provide a first glimpse [5], a dedicated Higgs factory will likely be required for exploring the compositeness of the Higgs boson. There are many options for composite Higgs models to choose from [6].

References

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