A Higgs Boson Composed of Gauge Bosons

F. J. Himpsel

Department of Physics, University of Wisconsin Madison,
1150 University Ave., Madison, WI 53706, USA, fhimpsel@wisc.edu

Abstract

It is proposed to replace the Higgs boson of the standard model by a Lorentz- and gauge-invariant combination of SU(2) gauge bosons. A pair of Higgs bosons is identified with pairs of gauge bosons by setting their mass Lagrangians proportional to each other. That immediately determines the mass of this composite Higgs boson. It becomes simply half of the vacuum expectation value of the standard Higgs boson and matches the observed mass with tree-level accuracy (2%). The two parameters of the standard Higgs potential are replaced by one-loop self-interactions of the SU(2) gauge bosons. The Brout-Englert-Higgs mechanism of spontaneous symmetry breaking is generalized from scalars to vectors. Their transverse components acquire finite vacuum expectation values which generate masses for the gauge bosons. This concept leads beyond the standard model by enabling calculations of the Higgs mass and its potential without adjustable parameters. It can be applied to non-abelian gauge theories in general, such as grand unified models and supersymmetry.

Contents

1. A Higgs Boson Composed of Gauge Bosons 2
2. Dynamical Symmetry Breaking via Gauge Boson Self-Interactions 6
3. Generalization of the Brout-Englert-Higgs Mechanism to Vector Bosons 9
4. Phenomenology 16
5. Summary and Outlook 20

References 21

Posted January 22, 2015
1. A Higgs Boson Composed of Gauge Bosons

The standard model [1] has been highly successful in describing the phenomenology of particle physics. It has passed many high precision tests with flying colors. But the intrinsic elegance of the electroweak gauge theory is blemished by the *ad-hoc* addition of the Higgs field. Rather than letting the gauge symmetry determine all the fundamental bosons, one has to justify the extra Higgs boson empirically. To make the situation worse, a term representing an imaginary mass is introduced into the Lagrangian of the Higgs field, together with a quartic term. Both are unheard of for Lagrangians of fundamental fields. These terms are inserted to obtain an attractive Higgs potential at small field amplitudes and a repulsive potential at large amplitudes. This combination is needed to generate a non-zero vacuum expectation value (VEV).

The discovery of a Higgs-like particle with a mass of about 126 GeV [2] does not alleviate these concerns about an *ad-hoc* Higgs scalar and its artificial potential. A possible escape from this dilemma is the notion of a composite Higgs boson, particularly if it is composed of known particles. In such models the gauge symmetry is broken dynamically by interactions between the constituents of the Higgs boson. A broad class of such models uses a condensate of fermion-antifermion pairs involving either known quarks or hypothetical techni-fermions [3],[4]. Since the Higgs-fermion interaction is proportional to the fermion mass in the standard model, the heaviest quarks are favored for dynamical symmetry breaking, such as top quark condensation [4]. These pairing models are able to produce masses for the top quark and the Higgs boson, but the masses come out too large — even when adjusting the inherent high-energy cutoff parameter \( \Lambda \).

The need for an energy parameter arises from a mismatch in dimensionality between the Higgs boson and a fermion pair. Bosons have dimension (mass)\(^1\) and fermions (mass)\(^{3/2}\), in units of \( \hbar, c \). Another cause for concern is the short lifetime of the top quark, which prevents the formation of bound states. This problem remains after adding the bottom quark to complete a SU(2) doublet [4].

The model proposed here involves pairing, too, but instead of fermion pairs we consider pairs of gauge bosons. Furthermore, the result of pairing is not an individual Higgs boson, but a pair of Higgs bosons. That guarantees a match of dimensions.
Figure 1 provides more specific heuristics for defining a composite Higgs boson, using diagrams from the standard model. In all three panels a pair of outgoing Higgs bosons on the left side is compared to pairs of outgoing SU(2) gauge bosons on the right. In (a) there are two incoming Higgs bosons, in (b) only one, and in (c) none. Removing the incoming Higgs bosons creates a relation between $H^2$ and $(Z^2+W^+W^-)$ in all three cases. Since the quadratic mass Lagrangians in (c) contain only pairs, they look attractive for defining a pair of composite Higgs bosons from pairs of gauge bosons.

\[
\begin{align*}
(a) & \quad H \times H = H \times Z + H \times W^+ \\
(b) & \quad H \times H = H \times Z + H \times W^- \\
(c) & \quad M_H^2 = M_Z^2 + M_W^2
\end{align*}
\]

**Figure 1** Standard model diagrams which suggest replacing a pair of outgoing Higgs bosons (in the 1st column) by pairs of outgoing SU(2) gauge bosons (in the 2nd and 3rd columns). The incoming particles are reduced from two in (a) to one in (b) and zero in (c). That suggests a definition of the composite Higgs boson via mass Lagrangians.

The concept of using gauge bosons as constituents is broadly applicable, since every gauge theory contains them. While they are Lorentz four-vectors, their scalar products match a pair of Higgs scalars. Calculations of the scattering amplitudes between gauge bosons indicate that their interaction is attractive when they form a Lorentz scalar and a singlet of the gauge symmetry (zero spin and isospin) [5]. The equivalence theorem [6] already connects longitudinal SU(2) gauge bosons with the Goldstone components of the complex Higgs doublet. Here we connect transverse SU(2) gauge bosons with the remaining Higgs component which corresponds to the observable Higgs particle.

Such considerations lead to the following strategy for replacing the Higgs boson of the standard model by a composite of SU(2) gauge bosons:
1) Eliminate the Higgs field from the Lagrangian of the standard model.
2) Define a composite Higgs boson from scalar products (= pairs) of gauge bosons.
3) Establish a potential for the gauge bosons via their one-loop self-interactions.
4) Minimize the potential to obtain symmetry-breaking VEVs for the gauge bosons.
5) Shift the fields by their VEVs to obtain gauge boson masses and self-interactions.

For defining a composite Higgs field $\Phi_0$ it is advisable to preserve the complex $SU(2)$ doublet structure of the Higgs field in the standard model. $\Phi_0$ can be written as a combination of a $SU(2)$ singlet $H_0$ and a triplet of Goldstone modes $(w_1, w_2, w_3)$:

$$\Phi_0 = \frac{1}{\sqrt{2}} \begin{bmatrix} w_2 + i w_1 \\ H_0 - i w_3 \end{bmatrix} \quad \Phi_0 = 1 \cdot H_0 + i \sum_i \tau_i \cdot w_i \quad \Phi_0 = \Phi_0 \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Phi_0 = \langle \Phi_0 \rangle + \Phi \quad \langle \Phi_0 \rangle = \frac{1}{\sqrt{2}} \langle H_0 \rangle \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$H_0 = \langle H_0 \rangle + H \quad \langle H_0 \rangle = \nu = 2^{-1/4} G_F^{-1/2} = 246.2 \text{ GeV}$$

The subscript zero indicates fields with finite VEVs. The complex doublet can be written as $2 \times 2$ matrix $\Phi_0$ which is defined via the Pauli matrices $\tau_i$ and the $2 \times 2$ unit matrix $1$. All $2 \times 2$ matrices are shown in bold. The singlet $H_0$ acquires a finite VEV $\langle H_0 \rangle = \nu$ via the Brout-Englert-Higgs mechanism [7], while the VEVs of the Goldstone modes vanish. The VEV $\nu$ is directly related to the experimental value of the four-fermion coupling constant $G_F$. After subtracting the VEVs from $\Phi_0, H_0$ one obtains the observable fields $\Phi, H$. The standard Higgs potential combines a quadratic and a quartic term:

$$V_\Phi = -\mu^2 \cdot \Phi_0^\dagger \Phi_0 + \lambda \cdot (\Phi_0^\dagger \Phi_0)^2 \quad \text{General gauge}$$

$$V_H = -\frac{1}{2} \mu^2 \cdot H_0^2 + \frac{1}{4} \lambda \cdot H_0^4 \quad \text{Unitary gauge}$$

These potentials are reduced from 4th order to 2nd order when using the pairs $\Phi_0^\dagger \Phi_0$ and $H_0^2$ as variables — again a hint that pairs may play a role in Higgs interactions.

The $SU(2)$ gauge bosons form a triplet $(W^1_\mu, W^2_\mu, W^3_\mu)$ that matches the Goldstone triplet. The sum of gauge boson pairs $\sum_i W^i_\mu W^{i*}_\mu$ is a Lorentz scalar and a $SU(2)$ singlet that matches $\Phi^\dagger \Phi$:

$$\Phi^\dagger \Phi = \frac{1}{2} \cdot [H^2 + \sum_i w_i^2] \propto -\frac{1}{2} \cdot \sum_i W^i_\mu W^{i*}_\mu$$

The minus sign ensures that the terms on both sides of the proportionality are positive, taking into account the space-like character of gauge bosons (in the $+---$ metric). The proportionality constant in (3) has yet to be determined, and the term $\sum_i W^i_\mu W^{i*}_\mu$ still lacks
gauge-invariance. These shortcomings are remediated by chiral electroweak Lagrangians [8]-[14]. These form gauge-invariant building blocks which also incorporate mixing between the SU(2) and U(1) gauge bosons (W^\pm_\mu and B_\mu). Even though they have been developed mainly for the heavy Higgs limit \( M_H \gg v \) (which is now unrealistic), they allow for a gauge-invariant generalization of \( \Sigma_i W^i_\mu W^i_\mu \). One starts with a nonlinear representation of the Goldstones \( w_i \) by casting them in the form of a SU(2) matrix \( U \):

\[
U = \exp(i \sum_i \tau^i_w \frac{w_i}{v})
\]

The SU(2) \( \times \) U(1) gauge bosons are then incorporated by defining the gauge-invariant derivative of the matrix \( U \):

\[
D_\mu U = \partial_\mu U - ig W^\mu_\mu U + ig' U B_\mu \quad \text{with} \quad W^i_\mu = \frac{1}{2} \tau^i W^\mu_\mu \quad B_\mu = \frac{1}{2} \tau^3 B_\mu
\]

Thereby the four gauge bosons \( W^\mu_\mu, B_\mu \) have been converted into the 2\( \times \)2 matrices \( W^i_\mu, B_\mu \).

The gauge-invariant derivative \( D_\mu \) of the matrix \( U \) defines a four-vector \( V_\mu \) which contains all four gauge bosons and their SU(2) \( \times \) U(1) couplings \( g, g' \):

\[
V_\mu = (D_\mu U) U^\dagger = i \cdot \frac{1}{2} \left[ \sum_i \tau^i \cdot (2 \partial_\mu W^i_\mu - g W^i_\mu) + \tau^3 g' B_\mu \right] = -V^\dagger_\mu
\]

In \( V_\mu \) the SU(2) gauge bosons \( W^i_\mu \) appear together with the derivatives of the Goldstones \( w_i \), showing again their close connection. Replacing \( \Sigma_i W^i_\mu W^i_\mu \) on the right side of (3) by the trace of \( V_\mu V^\mu \) establishes a gauge-invariant generalization which includes mixing of the SU(2) \( \times \) U(1) gauge bosons:

\[
\text{tr}[V_\mu V^\mu] = -g^2 \cdot \left[ (W^\mu_\mu - 2 \partial_\mu \frac{W^\mu_\mu}{c_{\mu}})(W^{-\mu}_\mu - 2 \partial^{-\mu}_\mu \frac{W^{-\mu}_\mu}{c_{\mu}}) + \frac{1}{2} (Z_\mu c_{\mu} - 2 \partial_\mu \frac{W^3_\mu}{c_{\mu}})(Z^{-\mu}_\mu c_{\mu} - 2 \partial^{-\mu}_\mu \frac{W^3_\mu}{c_{\mu}}) \right]
\]

\[
\rightarrow -\frac{1}{2} g^2 \cdot \sum_i W^i_\mu W^{i\mu}_\mu \quad \text{for the unitary gauge}
\]

\( W^\pm_\mu, Z_\mu \) and \( c_\nu^2 = \cos^2 \theta_\nu = g^2/(g^2 + g'^2) \) are defined as usual. Multiplication by \(-\frac{1}{2} v^2\) generates the tree-level mass Lagrangian for the gauge bosons:

\[
L_M^{ZW} = M_W^2 \cdot (W^+ W^-) + \frac{1}{2} M_Z^2 \cdot (ZZ)
\]

Scalar products have been abbreviated by parentheses. The photon does not appear with the SU(2) gauge bosons, because it is massless. Likewise, one can multiply \( \Phi^\dagger \Phi \) on the left side of (3) with the same factor \(-\frac{1}{2} v^2\) to obtain the Lagrangian for a scalar mass \( \frac{1}{2} v \). In the unitary gauge the Goldstones vanish, and one obtains:

\[
L_M^H = -\frac{1}{2} M_H^2 \cdot H^2
\]

\[\text{with} \quad M_H = \frac{1}{2} v \quad g = M_W/M_Z \]
The scalar mass is assigned to the tree-level mass of the composite Higgs boson. The resulting value \( M_H = \frac{1}{2} v = 2^{-5/4} G_F^{-1/2} = 123 \) GeV matches the observed Higgs mass of 126 GeV to about 2%. A comparable agreement exists between the tree-level mass of the W gauge boson \( M_W = \frac{1}{2} g v = 78.9 \) GeV in (8) and its observed mass of 80.4 GeV. Such an accuracy is typical of the tree-level approximation, which neglects loop corrections of the order \( \alpha_w = g^2/4\pi \approx 3\% \). It is reassuring to see the Higgs mass emerging directly from the concept of a Higgs boson composed of gauge bosons.

Next we establish a relation between pairs of gauge bosons and a pair of Higgs bosons by setting the quadratic mass Lagrangians (8) and (9) equal to each other:

\[
L^H_M = L^Z_W
\]

After dividing both sides by \(-(\frac{1}{2} v)^2\) one arrives at a simple, gauge-invariant relation:

\[
\Phi^\dagger \Phi = \text{tr}[V_\mu V^\mu] \quad \text{General gauge}
\]

\[
\frac{1}{2} H^2 = -g^2 \cdot [(W^+ W^-) + \frac{1}{2} (Z^+ Z^-)/c_w^2] \quad \text{Unitary gauge}
\]

Replacing \( H \) by \((H_0 - v)\) implies finite VEVs \( \langle W_0^\pm \rangle, \langle Z_0 \rangle \) for the gauge boson fields:

\[
W_0^\pm = \langle W_0^\pm \rangle + W^\pm \quad Z_0 = \langle Z_0 \rangle + Z
\]

Those VEVs have to be transverse to satisfy Lorentz- and gauge-invariance, as discussed in Section 3. The Lagrangian fields \( H_0, W_0^\pm, Z_0 \) then obey a more complicated relation:

\[
\frac{1}{2} (H_0^2 - 2vH_0) = -g^2 \cdot \{(W_0^+ W_0^-) + \frac{1}{2} (Z_0 Z_0)/c_w^2\} - \{(\langle W_0^+ \rangle W_0^-) + (W_0^+ \langle W_0^- \rangle) + (\langle Z_0 Z_0 \rangle/c_w^2)\}
\]

For the vacuum state, this becomes a relation between the VEVs of the Lagrangian fields:

\[
v^2 = g^2 \cdot (2w^2 + z^2/c_w^2) \quad w^2 = -\langle W_0^+ \rangle \langle W_0^- \rangle \quad z^2 = -\langle Z_0 \rangle \langle Z_0 \rangle
\]

2. Dynamical Symmetry Breaking via Gauge Boson Self-Interactions

To test whether the gauge bosons that make up the composite Higgs boson can cause dynamical symmetry breaking, it is useful to have a simple model potential. Such a potential can be constructed from 2nd and 4th order terms, like the Higgs potential in (2):

\[
V_V = -\mu^2 \cdot \text{tr}[(V_0 V_0)] + \lambda \cdot (\text{tr}[V_0 V_0])^2 \quad \text{General gauge}
\]

\[
V_V = \mu^2 g^2 \cdot \{(W_0^+ W_0^-) + \frac{1}{2} (Z_0 Z_0)/c_w^2\} + \lambda g^4 \cdot \{(W_0^+ W_0^-) + \frac{1}{2} (Z_0 Z_0)/c_w^2\} \quad \text{Unitary gauge}
\]
This is not the standard Higgs potential, though, since the relations (11),(12) become more complicated for the Lagrangian fields (compare (14)). The 4th order term of this gauge boson potential has the form of a well-known chiral electroweak Lagrangian [8]-[14]:

\[(18) \quad L_5 = \alpha_5 \cdot (\text{tr}[V_\mu V^\mu])^2 \rightarrow \alpha_5 \cdot g^4 \left[ (W^+ W^-) + \frac{1}{2} (Z Z)^2 / c_w^2 \right]^2 \quad \text{for the unitary gauge}\]

The gauge boson potential (17) is plotted versus the two gauge fields in Figure 2a. Thereby we have used the scalar products \((W_0^+ W_0^-)\) and \((Z_0 Z_0)\) to define the two field variables \(w_0 = \left[-(W_0^+ W_0^-)\right]^{1/2}\) and \(z_0 = \left[-(Z_0 Z_0)\right]^{1/2}\). Similar pair products appeared already in the definition (12) of the composite Higgs boson. The topography of the model potential is rather peculiar, since the minimum is stretched out over a line. A unique minimum has been generated in Figure 2b by reducing the term \((W_0^+ W_0^-)(Z_0 Z_0)\) by a factor \(10^{-12}\) and increasing the terms \((W_0^+ W_0^-)^2\) and \((Z_0 Z_0)^2\) by a factor \(10^{11}\).

![Figure 2](image)

**Figure 2** The potential of the gauge bosons which make up the composite Higgs boson, plotted versus the gauge boson amplitudes \(w_0, z_0\). (a) is for the model potential (17) which forms a flat potential valley. (b) is for slightly-modified coefficients which produce a unique minimum. The horizontal axes are in GeV, the vertical axis is in \((\text{GeV})^4\).

The two model potentials in Fig. 2 demonstrate that scalar gauge boson pairs of the form \((W_0^+ W_0^-)\) and \((Z_0 Z_0)\) are able to develop finite VEVs. That makes them capable of spontaneous symmetry breaking. A composite of such gauge boson therefore is able to mimic a pair of Higgs scalars. But these model potentials still contain the two Higgs parameters \(\mu^2, \lambda\). The ultimate goal is a derivation of the gauge boson potential from the quadratic and quartic self-interactions, which are free of adjustable parameters.
Before tackling this task it is helpful to briefly review the standard formalism for converting the Lagrangian Higgs field $H_0$ and its potential to the observable Higgs boson $H$, including its mass term and its self-interactions. Figure 3 visualizes the next step, where the VEV $v$ is extracted from the Lagrangian Higgs field $H_0$:

$$V_H = -\frac{1}{2} \mu^2 \cdot H_0^2 + \frac{1}{4} \lambda \cdot H_0^4$$

$$H_0 \to (v + H),$$

$$= \frac{1}{2} M_H^2 \cdot \left[ -\frac{1}{4} v^2 + H^2 + v^{-1} \cdot H^3 + \frac{1}{4} v^{-2} \cdot H^4 \right]$$

$$\mu^2 \to \frac{1}{2} M_H^2 \quad \lambda \to \frac{1}{2} M_H^2 / v^2$$

$$= -\frac{1}{2} M_H^2 + \frac{1}{2} M_H^2 \cdot H^2 + \frac{1}{4} M_H \cdot H^3 + \frac{1}{8} v^2 \cdot H^4$$

$$\nu \to 2 M_H$$

The quadratic coefficient changes its sign and magnitude from $-\frac{1}{2} \mu^2$ to $+\mu^2$. Instead of the negative dashed parabola at $H_0=0$ one has the positive dotted parabola at $H=0$. That is the mass term of the observable Higgs boson. The extra cubic term consists of a mixed product between $H_0$ and its VEV $v$. The second line in (19) has been written in terms of the observables $M_H, v$. These are given by:

$$M_H^2 = 2 \cdot \mu^2$$

$$v = \mu / \sqrt{\lambda}$$

The parameters $\mu^2, \lambda$ are obtained by inverting (20) and inserting the experimental results $M_H = 125.7 \text{ GeV}$ and $v = 2^{-1/4} G_F^{-1/2} = 246.2 \text{ GeV}$ from [2]:

$$\mu^2 = \frac{1}{2} M_H^2 = (88.9 \text{ GeV})^2$$

$$\lambda = \frac{1}{2} M_H^2 / v^2 = 0.130$$

This information leads to the quantitative plot of the Higgs potential in Fig. 3. The last line in (19) goes beyond the standard model by introducing our result (9) for the mass of the composite Higgs boson.

**Figure 3** Plot of the standard Higgs potential $V_H(H_0)$ in (2). The two parameters $\mu^2$ and $\lambda$ are obtained from the observed Higgs mass $M_H$ and the VEV $v$ of $H_0$ via (1),(21). The origin of the observable Higgs boson $H = H_0 - v$ is indicated. $V_H$ has the dimension (mass)$^4$, since it is part of a Lagrangian. In the composite model the dashed potential originates from the gauge boson self-energy.
3. Generalization of the Brout-Englert-Higgs Mechanism to Vector Bosons

To construct the potential for gauge bosons we select one-loop self-interactions containing the neutral pairs \((W^0 W^0)\) or \((Z^0 Z^0)\) as external lines. The choice of the Lagrangian fields \(W^\pm, Z^0\) is motivated by the fact that the scalar Higgs boson potential is part of the Lagrangian. Had we chosen the observable gauge bosons \(W^\pm, Z\) we would have encountered odd powers of the fields in the Lagrangian analogous to the \(H^3\) term in the Higgs potential (19). Those originate from products of \(W^\pm, Z\) with VEVs,

One-loop diagrams of \(O(g^2)\) and \(O(g^4)\) are shown and in Figure 4 for the standard model and in Figures 5, 6 for a pure SU(2) model (which is much easier to handle). The subscripts 0 for Lagrangian fields have been suppressed. The 2\(^{nd}\) and 4\(^{th}\) order terms can be viewed as are the first two terms of an infinite series whose \(N^{th}\) term is represented by Feynman diagrams of \(O(g^{2N})\) with \(N\) pairs of external gauge bosons connected to a loop. The next level of accuracy would involve 6\(^{th}\) and 8\(^{th}\) order terms, assuming that the trend of alternating signs for even and odd \(N\) continues.

The Lagrangian fields \(W^\pm, Z^0\) have large VEVs, around which the observable fields \(W^\pm, Z\) oscillate with small amplitudes. For applying perturbation there are two options to consider:

1) Use a truncated perturbation series in the weak fields \(W^\pm, Z\). In this case one needs to include additional diagrams that combine the fields \(W^\pm, Z\) with VEVs, such as the triple gauge boson couplings \(W^+ W^- Z\) and \(ZZZ\). Such terms are worked out in (26) below. They are to be distinguished from the triple SU(2) gauge couplings, which contain a derivative instead of a VEV.

2) Sum the one-loop series for the strong fields \(W^\pm, Z^0\) analytically to infinity. That has been achieved for the quartic Higgs self-interaction [15]. This looks like a formidable task for the gauge bosons in the standard model, where the number of diagrams escalates rapidly with the number of external lines. But it might be possible for a pure SU(2).

The following figures show irreducible diagrams in the unitary gauge. A complete set of diagrams for a general gauge would have to include reducible diagrams, crossed diagrams, Goldstone modes, gauge fixing terms, Fadeev-Popov ghosts, and counterterms for renormalization.
Figure 4  Irreducible one-loop self-interactions of the SU(2) gauge bosons in the standard model: (a) Quadratic diagrams of $O(g^2)$ for the self-energies $\Sigma^W$ and $\Sigma^Z$, (b)-(e) diagrams of $O(g^4)$ representing scattering between the neutral gauge boson pairs $(W_0^- W_0^+) \text{ and } (Z_0^- Z_0^+)$. They generate the quadruple vertex corrections $A^{W,W}, A^{W,Z}, A^{Z,Z}$. Left-handed fermion doublets are labeled $(\psi, \psi')$. 
Figure 5  Quadratic self-interactions of $O(g^2)$ for the pure SU(2) gauge bosons $W_i$: (a) gauge boson loop with a quadruple vertex, (b) gauge boson loop with two triple vertices, (c) fermion loop. Together they represent the gauge boson self-energy $\Sigma$. Such diagrams form the attractive part of the potential.

Figure 6  Quadruple self-interactions of $O(g^4)$ between pure SU(2) gauge bosons, forming the repulsive part of the potential: (a) two quadruple vertices, (b) one quadruple vertex plus two triple vertices, (c) four triple vertices, (d) four vertices with fermions. These diagrams describe the vertex correction $\Lambda$ for scattering between pairs of equal gauge bosons. $\psi$ represents a fermion.

From the combinations of external lines in Figure 4 one obtains the general structure of the dynamic gauge boson potential:

$$V_{\text{dyn}} = V_{2V} + V_{4V}$$

$$V_{2V} = -\Sigma^W \cdot (W_0^2 W_0^2) - \frac{1}{2} \Sigma^Z \cdot (Z_0 Z_0)$$

$$V_{4V} = \Lambda^{WW} \cdot (W_0^2 W_0^2)^2 + \Lambda^{WZ} \cdot (W_0^2 W_0^2)(Z_0 Z_0) + \frac{1}{4} \Lambda^{ZZ} \cdot (Z_0 Z_0)^2$$

The factor $\frac{1}{2}$ with $(Z_0 Z_0)$ has been included to ensure that $\Sigma^Z$ adds to $M^2_Z$ in the mass Lagrangian (8). It does not occur with $(W_0^2 W_0^2)$, which represents the two real fields $W^1, W^2$. The same prefactors were kept in defining $V_{4V}$. The model potential (17) corresponds to the coefficients:

$$\Sigma^W = c_w^2 \cdot \Sigma^Z = -\mu^2 g^2$$

$$\Lambda^{WW} = c_w^4 \cdot \Lambda^{WZ} = c_w^4 \cdot \Lambda^{ZZ} = \lambda g^4$$

With $\mu^2, \lambda$ obtained from $M_W, v$ via (21) one arrives at the following coefficients:

$\Sigma^W = -(57.0 \text{ GeV})^2$  \ $\Sigma^Z = -(64.6 \text{ GeV})^2$  \ $\Lambda^{WW} = 0.0219$  \ $\Lambda^{WZ} = 0.0282$  \ $\Lambda^{ZZ} = 0.0363$

The characteristics of the gauge boson potential come out more clearly by plotting Figure 2 versus the pair variables $w_0^2 = -(W_0^2 W_0^2)$ and $z_0^2 = -(Z_0 Z_0)$, as done in Figure 7. These variables are matched to the scalar products of gauge bosons occurring in the
gauge boson potential (22). 4th order terms now become quadratic and 2nd order terms linear. Since the potential has been reduced to a quadratic form, it can be analyzed in terms of quadric surfaces in the three-dimensional space spanned by the variables \( x = z_0^2 \), \( y = w_0^2 \), and \( z = V_{\text{dyn}} \). The potential surfaces have paraboloidal character, since the variable \( z \) appears only linearly, not quadratically. For dynamical symmetry breaking one needs a potential surface with a minimum, which has the general form of an elliptic paraboloid as shown in Fig. 7b. For the model potential in Section 2 the paraboloid degenerates to a parabolic cylinder (Fig. 7a). The topology of these potential surfaces is determined by the determinant of the coefficient matrix for the 2nd order terms in (22).

![Figure 7](image-url)

**Figure 7**  Plots of the two gauge boson potentials from Fig. 2, using the squared amplitudes \( w_0^2, z_0^2 \) as variables instead of \( w_0, z_0 \). That leads to a simpler topography of the potential. In (a) the extended minimum of the model potential (17),(23) becomes a straight line at the bottom of a parabolic cylinder when plotted against \( w_0^2, z_0^2 \). For the modified potential in (b) the potential surface becomes an elliptical paraboloid. This simplicity matches the concept of gauge boson pairs as natural variables of the symmetry-breaking potential. The horizontal axes are now in \((\text{GeV})^2\).

The next step consists of minimizing the gauge boson potential with respect to the two (positive) variables \( -(W_0^2W_0) = w_0^2 \) and \( -(Z_0Z_0) = z_0^2 \). If a well-defined minimum exists at finite \( w_0^2 = w^2 \) and \( z_0^2 = z^2 \), the corresponding VEVs are subtracted from \( W_0, Z_0 \) to obtain the observable fields \( W^\pm, Z \). To be attractive for weak fields and repulsive for strong fields, the potential needs negative quadratic coefficients \( \Sigma^W, \Sigma^Z \) and positive quartic coefficients \( \Lambda^{WW}, \Lambda^{ZZ} \) (analogous to the scalar Higgs potential). This is not
somewhat hidden in the quadratic term $V_{2V}$ which contains a product of three minus signs. One is explicit, the other two are implicit because $\Sigma^W, \Sigma^Z$ and the scalar products of vector bosons are negative.

At the minimum of $V_{\text{dyn}}(w_0^2, z_0^2)$ the partial derivatives with respect to the pair coordinates $w_0^2, z_0^2$ have to vanish. To obtain the proper topology one has to consider the determinant $\frac{1}{4}[\Lambda^{WW}, \Lambda^{ZZ} - (\Lambda^{WZ})^2]$ of the 2×2 matrix containing the 2nd order coefficients $\Lambda^{WW}, \frac{1}{4}\Lambda^{ZZ},$ and $\frac{1}{2}\Lambda^{WZ}.$ The degenerate case with vanishing determinant will be postponed to Section 4. For a non-vanishing, positive determinant one obtains a single minimum at the VEVs $w^2, z^2$ under the following conditions:

\begin{align*}
(24a) & \quad \partial V_{\text{dyn}}/\partial (w_0^2) = 0 \quad \partial V_{\text{dyn}}/\partial (z_0^2) = 0 \\
(24b) & \quad \Sigma^W, \Sigma^Z < 0 \quad \Lambda^{WW}, \Lambda^{ZZ} > 0 \\
(24c) & \quad \Lambda^{WW}, |\Sigma^z| > \Lambda^{WZ}, |\Sigma^w| \quad \Lambda^{ZZ}, |\Sigma^w| > \Lambda^{WZ}, |\Sigma^z| \quad \Lambda^{WW}, \Lambda^{ZZ} > (\Lambda^{WZ})^2 \\
(24d) & \quad w^2 = -((W_0^+)(W_0^-)) = \frac{1}{2}(\Lambda^{ZZ}, |\Sigma^z| - \Lambda^{WZ}, |\Sigma^w|)/[\Lambda^{WW}, \Lambda^{ZZ} - (\Lambda^{WZ})^2] \\
& \quad z^2 = -((Z_0^+)(Z_0^-)) = (\Lambda^{WW}, |\Sigma^w| - \Lambda^{WZ}, |\Sigma^z|)/[\Lambda^{WW}, \Lambda^{ZZ} - (\Lambda^{WZ})^2]
\end{align*}

When assigning a VEV to a vector boson, one has to be careful to preserve the Lorentz invariance of the vacuum. It would be violated by choosing a fixed vector in space-time for the VEV. But it is possible to escape this dilemma by assigning the VEV to the longitudinal or transverse components of a vector boson. Thereby the VEV of each particle is oriented relative to its momentum vector. Since the infinite density of virtual particles in the vacuum of quantum field theory comprise the full range of momenta, individual orientation effects are averaged out.

At a first glance, one might be tempted to choose the longitudinal component of a vector boson for its VEV. That exists only when the symmetry is broken and the gauge boson becomes massive. But such an assignment would be gauge-dependent, since the longitudinal component can be eliminated using the Landau gauge. Its role is transferred to a Goldstone mode whose VEV vanishes. Thereby the VEV of the gauge boson is gauged away. Consequently we associate the VEV of a gauge boson with its two transverse components. By choosing the momentum of a gauge boson as the z-axis of a local reference frame, one can convert the vector bosons $W_0^\pm, Z_0$ into scalars that are multiplied by one of the two transverse polarization vectors $\epsilon^T_{\mu\nu}.$ These scalars are
identified with the field amplitudes \(w_0, z_0\) that were already used for plotting the potentials. In the Landau gauge one obtains:

\[
\begin{align*}
W_0^\pm &= \langle W_0^\pm \rangle + W^\pm \\
Z_0 &= \langle Z_0 \rangle + Z \\
W_0^1 &= \langle W_0^1 \rangle + W^1 \\
W_0^2 &= \langle W_0^2 \rangle + W^2
\end{align*}
\]

\[
W^\pm = w^\pm \cdot \varepsilon_{T,n}
\]

\[
Z = z \cdot \varepsilon_{T,n}
\]

\[
w^+ = (w^1 - iw^2)\sqrt{2} \\
w^- = (w^1 + iw^2)\sqrt{2}
\]

The VEV of the photon remains zero, since the U(1) symmetry of QED is not broken.

With the structure of the VEVs in hand, one can convert the Lagrangian fields \(W_0^\pm, Z_0\) to the observable fields \(W^\pm, Z\). The required substitutions \(W_0^\pm \rightarrow (\langle W_0^\pm \rangle + W^\pm)\), \(Z_0 \rightarrow (\langle Z_0 \rangle + Z)\) were anticipated in (13). Products of fields and VEVs generate odd powers of the fields, the same way as the cubic term for the scalar Higgs boson in (19):

\[
\begin{align*}
V_{\text{dyn}} &= \Lambda_{WW} \cdot (w^+ w^-)^2 + \Lambda_{WZ} \cdot (w^+ w^-)(zz) + \frac{1}{4} \Lambda_{ZZ} \cdot (zz)^2 \\
&+ 2w \Lambda_{WW} \cdot (w^+ w^-)(w^1 + w^2) + 2z \Lambda_{WZ} \cdot (w^+ w^-) z + w \Lambda_{WZ} \cdot (w^1 + w^2)(zz) + z \Lambda_{ZZ} \cdot (zz) z \\
&+ 2w^2 \Lambda_{WW} \cdot (w^+ w^-) + 2u \Lambda_{WW} \cdot (w^1 w^2) + 2u \Lambda_{WZ} \cdot (w^1 + w^2) z + z^2 \Lambda_{ZZ} \cdot (zz) \\
&+ \frac{1}{2} w^2 \Sigma^W + \frac{1}{4} z^2 \Sigma^Z
\end{align*}
\]

The three scalar fields \(w^\pm, z\) describe the transverse components of the observable gauge bosons, as laid out in (25). The terms can be simplified using the transverse components of the neutral gauge bosons \(w^1 = (w^+ + w^-)\sqrt{2}\) and \(w^2 = i(w^+ - w^-)\sqrt{2}\) as variables. This result is valid in the Landau gauge, where the longitudinal components have been converted to Goldstone modes. Consequently one has to use the gauge-invariant definition of the composite Higgs boson from (7),(11),(13), which includes Goldstones.

The constant term in (26) corresponds simply to a change of the zero point energy, which does not matter here. The linear terms vanish, since the origin of the shifted fields lies at the minimum of the potential. The coefficients of the quadratic terms \((w^+ w^-), (zz)\) in the 3\textsuperscript{rd} row of (26) provide the gauge boson masses via the mass Lagrangians (8) and the definitions \(w_0^ \pm = -(W_0^ 0 W_0^), \ z_0^2 = -(Z_0 Z_0)\):

\[
\begin{align*}
M_W^2 &= 2 \Lambda_{WW} \cdot w^2 \\
M_Z^2 &= 2 \Lambda_{ZZ} \cdot z^2 \\
c_w &= M_W/M_Z = (\Lambda_{WW}/\Lambda_{ZZ})^{1/2} \cdot w/z
\end{align*}
\]
These mass terms are analogs of the result $M_\text{H}^2 = 2\lambda \cdot \nu^2$ for the standard Higgs boson. The VEVs $w^2, z^2$ can be expressed in terms of the coefficients for the gauge boson potential using (24d). The electroweak mixing parameter becomes $c_w = M_W/M_Z$ in the on-shell renormalization scheme. The expressions for $M_W^2, M_Z^2$ in (27) are indeed positive with the constraints $\Lambda^{WW}, \Lambda^{ZZ} > 0$ from (24b). The remaining terms in (26) represent self-interactions of the gauge bosons, analogous to the cubic and quartic Higgs self-interactions.

A quantitative test of this model requires calculations of the gauge boson self-interactions $\Sigma^W, \Sigma^Z$ and $\Lambda^{WW}, \Lambda^{WZ}, \Lambda^{ZZ}$. The self-energies $\Sigma^W, \Sigma^Z$ have been obtained in various places [16],[17], but renormalized results are difficult to find in explicit form. A plot of the renormalized transverse self-energies by Böhm et al. 1986 [16] shows that the gauge boson self-energies are negative, as required for an attractive potential that induces spontaneous symmetry-breaking. But the mass of the top quark was highly underestimated at that time, making the results only qualitative. Another interesting finding in this work is a strong increase of the gauge boson self-energy when going away from the mass shell towards higher energies. It remains approximately quadratic up to a few 100 GeV. That corresponds to the VEVs $w, z$ of the gauge bosons, which determine the mass scale at which the self-energies $\Sigma^W, \Sigma^Z$ need to be evaluated for determining the gauge boson masses. Thus, even moderate radiative corrections can give rise to substantial masses.

Calculations of the quadruple self-interactions $\Lambda^{WW}, \Lambda^{WZ}, \Lambda^{ZZ}$ are still lacking, even though many results have been published for chiral electroweak Lagrangians in the heavy Higgs limit [8]-[13]. They have become inappropriate after the 2012 discovery of a light Higgs particle [2]. More recent approaches [14] take the light Higgs boson into account and thus should be applicable. There has been extensive work on invariant amplitudes of quadruple vertices encountered in boson-boson scattering. The next-to-leading amplitudes of $O(g^4)$ are related to the one-loop coefficients $\Lambda^{WW}, \Lambda^{WZ}, \Lambda^{ZZ}$. But most calculations have been performed in the high energy limit, spurred by the unitarity catastrophe looming at the TeV energy scale for a heavy Higgs boson [6]. In that limit the longitudinal components of the gauge bosons dominate. The transverse components
were neglected. Exact amplitudes have been calculated for $WW \rightarrow WW$ and $ZZ \rightarrow ZZ$ scattering at the one-loop level, involving hundreds of diagrams [18]. It would be interesting to see whether these amplitudes can be translated into the coefficients $\Lambda^{WW}$ and $\Lambda^{ZZ}$ for the gauge boson potential.

For further guidance one can consult long-standing efforts to generate the quadratic coefficient $\mu^2$ of the Higgs potential from its quartic self-interaction [15], [19], [20]. Calculations carried out within the standard model via the renormalization group equations have found expressions for the Higgs self-energy $\Sigma^H$ of the form:

$$
\Sigma^H(\Lambda) = 3(\Lambda/4\pi v)^2 \cdot [M^2_{WW}(\Lambda) + 2M^2_{ZZ}(\Lambda) + M^2_{ZZ}(\Lambda) - 4m^2_t(\Lambda)] < 0
$$

For energy scales $\Lambda$ ranging from $v$ all the way up to about $10^{17}$ GeV the result is indeed negative [20], as required for spontaneous symmetry breaking. The sign is dominated by the contribution from the mass $m_t$ of the top quark. It is interesting to notice that the negative self-energies of the gauge bosons are also dominated by fermion loops [16], [17]. But in that case the light fermions dominate.

4. Phenomenology

In the absence of calculations for the five coefficients of the gauge boson potential (22) one can at least test whether they can be adjusted to reproduce observables, such as $M_W, M_Z, M_H, v$, and the mixing ratio $\tan\theta_H = 1/2\cos^2\beta$ between $(W^+W^-)$ and $(ZZ)$ in the definition (12) of the composite Higgs boson. The Higgs VEV $\nu$ is related to the four-fermion coupling $G_F$ via (1). While the previous section was aimed at calculating observables from the coefficients, the goal is now to determine the coefficients from observables. For this purpose we characterize the potential surfaces that can be obtained with various parameter sets. Contour plots, such as those in Figure 8, are a good way to analyze the situation. In general, one can use the three inequalities in (24c) as guidelines for the shape of the contours. They affect the signs of the numerators and denominators in (24d) which in turn determine the gauge boson VEVs $w^2, z^2$. If the inequalities are satisfied, one obtains a well-defined minimum in the $w_0^2, z_0^2$ plane (top left panel). If one has equalities instead, the minimum becomes stretched out into a line (central panel). With a negative numerator one of the VEVs gets pushed to the boundary of the allowed
quadrant and vanishes (bottom panels). If the common denominator $[\Lambda^{WW}\Lambda^{ZZ}-(\Lambda^{WZ})^2]$ in (24d) turns negative, the minimum becomes a saddle point and drives both VEVs towards the boundaries (upper right panel).

![Figure 8](image)

**Figure 8** The contours of the gauge boson potential (22) as a function of the pair coordinates $w_0^2, z_0^2$ [in (GeV)$^2$]. Minima are shown dark. The panels illustrate various scenarios in terms of the three inequalities (24c). The central panel is for the degenerate model potential (17),(23) with $\Lambda^{WW}\Lambda^{ZZ} = (\Lambda^{WZ})^2$, where the minimum expands into a line. The upper left panel for $\Lambda^{WW}\Lambda^{ZZ} > (\Lambda^{WZ})^2$ exhibits a unique minimum with finite VEVs. The opposite inequality $\Lambda^{WW}\Lambda^{ZZ} < (\Lambda^{WZ})^2$ leads to a saddle point (upper right). That creates a bistable situation with two minima at the edges of the allowed region ($w_0^2, z_0^2 \geq 0$). When one of the other inequalities in (24c) is violated, a single minimum tends to occur at one of the two edges, i.e., one of the VEVs vanishes (bottom panels).

The degenerate case shown at the center of Figure 8 is easier to analyze, because the number of coefficients is reduced from five to three. The relations (24a-d) for the non-degenerate case are replaced by the following set:

(29a) $\partial V^\text{dyn}/\partial (w_0^2) = 0$ \hspace{1cm} $\partial V^\text{dyn}/\partial (z_0^2) = 0$

(29b) $\Sigma^W, \Sigma^Z < 0$ \hspace{1cm} $\Lambda^{WW}, \Lambda^{ZZ} > 0$ \hspace{1cm} $\Lambda^{WZ} > 0$

(29c) $\Lambda^{WW} \Sigma^Z = \Lambda^{WZ} \Sigma^W$ \hspace{1cm} $\Lambda^{ZZ} \Sigma^W = \Lambda^{WZ} \Sigma^Z$

(29d) $w^2 = -\frac{1}{2} \Sigma^Z /[\Sigma^W/(\Lambda^{WZ} + z^2)]$ \hspace{1cm} $\tan(\theta_H) = -d(w^2)/d(z^2) = \frac{1}{2} \Sigma^Z/\Sigma^W$

Equation (29d) describes a line of possible VEVs $w^2,z^2$ in the $w_0^2,z_0^2$ plane (labeled VEV in Figure 9). This characteristic of degenerate potentials allows extra freedom in satisfying experimental constrains compared to a point-like minimum. The endpoints of the VEV line define the slope $\tan(\theta_H)$. 

17
Figure 9  Analysis of the degenerate potentials (29a-d). The full line covers possible locations of the VEV in the \(w_0^2, z_0^2\) plane, with \(w_0^2 = -(W_0^2 - W_0^2)\) and \(z_0^2 = -(Z_0 Z_0)\). The arrow for the Higgs pair \(H^2\) shows the mixture (12) of the gauge boson pairs \(w^2 = -(W^+ W^-)\) and \(z^2 = -(Z Z)\). Two-parameter potentials have \(\theta_H\) fixed by \(\tan(\theta_H) = 1/2c_w^2\).

For an overview of possible scenarios we consider the number of available parameters. For the non-degenerate potential in Section 3 all five coefficients \(\Sigma^W, \Sigma^Z, \Lambda^{WW}, \Lambda^{WZ}, \Lambda^{ZZ}\) are independent. For degenerate potentials they are constrained by the two independent constraints contained in (29c). They can be used to eliminate \(\Lambda^{WW}, \Lambda^{ZZ}\):

\[
\Lambda^{WW} = \Lambda^{WZ} \cdot (\Sigma^W/\Sigma^Z) \quad \Lambda^{ZZ} = \Lambda^{WZ} \cdot (\Sigma^Z/\Sigma^W)
\]

By adding a third constraint one arrives at the model potential (17) from Section 2:

\[
\Sigma^Z = \Sigma^W/c_w^2 \quad c_w^2 = M_W^2/M_Z^2 = 0.777
\]

The two remaining coefficients \(\Sigma^W, \Lambda^{WZ}\) can be mapped onto the two parameters \(\mu^2, \lambda\) of the standard Higgs potential via (23). The slope \(\tan(\theta_H)\) of the VEV line is now fixed by the relation (15). Its endpoints are (using \(g = (4\pi\alpha s_w^2)^{1/2}\), \(s_w^2 = 1 - c_w^2\)):

\[
w_0^2 = \frac{1}{2}(v/g)^2 = (272 \text{ GeV})^2 \quad z_0^2 = (c_w v/g)^2 = (339 \text{ GeV})^2 \quad g = 0.641
\]

Figure 9 illustrates this case, with the two parameters chosen to reproduce the experimental values of \(M_{H^0}, v\) via (21). This figures consists of two separate pieces. The upper right quadrant shows observable fields and the remainder is for Lagrangian fields (with subscript 0). They are connected at the VEV \(w^2, z^2\) (small circle) but are otherwise related in a more complicated way (compare (12) and (14)). Strictly speaking, one has to deal with the three coordinates \((w^1)^2, (w^2)^2, \text{ and } z^2\), but the first two are equivalent.

The arrow \(H^2\) for observable Higgs pairs has its origin at the VEV \(w^2, z^2\). From the definition of the composite Higgs boson in (12) one can view \(H^2\) as a linear function of the gauge boson pairs \(w^2 = -(W^+ W^-)\) and \(z^2 = -(Z Z)\). The VEV line is an equipotential contour and therefore orthogonal to the arrow for the mass eigenstate \(H^2\). The latter can be decomposed into a rotation by the angle \(\theta_H\) from the \(w^2\) axis, combined with a multiplication by the scale factor \(s\).
\[
H^2 = -g^2 \cdot [2(W^+W^-) + (ZZ)/c_w^2] = -s \cdot [\cos \theta_H \cdot (W^+W^-) + \sin \theta_H \cdot (ZZ)]
\]

\[
\begin{align*}
\cos \theta_H &= \frac{2g^2}{s} \quad & \tan \theta_H &= \frac{1/2c_w^2}{s} = 0.6435 \quad & \theta_H &= 32.8^0 \\
\sin \theta_H &= \frac{g^2}{c_w^2 s} \quad & s &= g^2(4+c_w^4)^{1/2} = 0.977
\end{align*}
\]

The Higgs mixing angle \( \theta_H \) is an analog of the electroweak mixing angle \( \theta_w \), but it mixes pairs instead of individual bosons. \( \theta_H \) and \( \theta_w \) are indeed related, since \( c_w^2 = \cos^2 \theta_w \).

Next we move on to three-parameter potentials by relaxing the condition (31). The rotation angle \( \theta_H \) in Fig. 9 is now adjustable. \( \Sigma^W, \Sigma^Z \) become independent and can be used to match the masses \( M_W^2, M_Z^2 \) via dynamical symmetry breaking. And \( \Lambda^{WZ} \) can be fixed by the Higgs boson VEV via (15). The resulting set of parameters becomes:

\[
\begin{align*}
\Sigma^W &= -(97.4 \text{ GeV})^2 \\
\Sigma^Z &= -(114.1 \text{ GeV})^2 \\
\Lambda^{WZ} &= 0.0632 \\
\Lambda^{ZZ} &= 0.0865 \\
\end{align*}
\]

Three independent parameters are not sufficient to correctly describe all the properties of the composite Higgs boson. With this parameter set the Higgs mass and \( \tan \theta_H \) come out incorrectly. That leads us towards non-degenerate potentials with five parameters.

Figure 10  Similar to Fig. 9, but for the case of non-degenerate potentials. The dashed line to the VEV \( w_0^2, z_0^2 \) is generally not parallel to the direction of the observable mass eigenstate \( H^2 \), which is dictated by (12). A second massive state appears parallel to the long axis of the elliptical potential contours (see Fig. 8, top left panel).

Non-degenerate potentials are shown in the upper left panel of Figure 8 and analyzed in Figure 10. Their five coefficients make the analysis more complicated, but also have a better chance to match the observables \( M_W, M_Z, M_H, \nu, \) and \( \tan \theta_H \). In contrast to the degenerate case in Figure 9, the line of possible VEVs shrinks to a well-defined minimum, surrounded by elliptical equipotential contours (Fig. 8). Since the observable Higgs boson \( H \) is a mass eigenstate, \( H^2 \) has to be aligned with the principal axes of the ellipses in the \( w^2, z^2 \) plane (gray crosshairs in Fig. 8 and arrow in Fig. 10). As a result one encounters two angles, \( \theta_H \) for the direction of \( H^2 \) in the \( w^2, z^2 \) plane and \( \theta_V \) for the location of the VEV in the \( w_0^2, z_0^2 \) plane (dashed line in Fig. 10).
An additional complication is the appearance of a second massive scalar along the other principal axis of the potential contours. This eigenstate combines WW and ZZ pairs with opposite signs, pointing towards an iso-tensor. In that case one expects a repulsive interaction [5] which would prevent pair formation. In the degenerate case the second eigenstate is not a problem, since the mass vanishes when the long axis of the elliptical contour approaches infinity. This mode does not contribute to the mass Lagrangians in (12) which define the composite Higgs boson.

The general method of handling the non-degenerate case has been outlined in Section 3, but a match of the five parameters to the five observables remains to be demonstrated. The following parameters match $M_W, M_Z, \nu, \tan\theta_H$, but not $M_H$:

$$\Sigma_W = -(95.5 \text{ GeV})^2 \quad \Sigma_Z = -(107.2 \text{ GeV})^2 \quad \Lambda^{WW} = 0.0644 \quad \Lambda^{WZ} = 0.0788 \quad \Lambda^{ZZ} = 0.1140$$

In any case one has to include longitudinal gauge bosons and their self-interactions when using relations that are valid in the unitary gauge, such as (12). In the Landau gauge these are traded for terms involving derivatives of the Goldstones (compare (7),(11)).

5. Summary and Outlook

In summary, a new concept is proposed for electroweak symmetry breaking, where the Higgs boson is identified with a scalar combination of gauge bosons in gauge-invariant fashion. That explains the mass of the Higgs boson with 2% accuracy. In order to replace the standard Higgs scalar, the Brout-Englert-Higgs mechanism of symmetry breaking is generalized from scalars to vectors. The ad-hoc Higgs potential of the standard model is replaced by self-interactions of the SU(2) gauge bosons, which can be calculated without adjustable parameters. Dynamical symmetry breaking then leads to finite VEVs of the transverse gauge bosons, which in turn generate gauge boson masses and self-interactions. Since gauge bosons and their interactions are connected directly to the symmetry group of a theory via the adjoint representation and gauge-invariant derivatives, the proposed mechanism of dynamical symmetry breaking is applicable to any non-abelian gauge theory, including grand unified theories and supersymmetry.

In order to test this model, the gauge boson self-interactions need to be worked out. These are the self-energies $\Sigma_W, \Sigma_Z$ and the four-fold vertex corrections $\Lambda^{WW}, \Lambda^{WZ}, \Lambda^{ZZ}$.
The VEV of the standard Higgs boson — which generates masses for the gauge bosons and for the Higgs itself — gets replaced by the VEVs of the $W^\pm$ and $Z$ gauge bosons. Since the standard Higgs boson interacts with most of the fundamental particles, its replacement implies rewriting a large portion of the standard model. Approximate results may be obtained by calculating gauge boson self-interactions within the standard model, assuming that the contribution of the standard Higgs boson is small for low-energy phenomena. The upcoming high-energy run of the LHC offers a great opportunity to test the characteristic couplings of the composite Higgs boson, as well as the new gauge boson couplings introduced by their VEVs. If confirmed, the concept of a Higgs boson composed of gauge bosons would open the door to escape the confine of the standard model and calculate previously inaccessible masses and couplings, such as the Higgs mass and its couplings $\mu^2$ and $\lambda$.

References


