Self-Similar Distribution Functions in the Solar Wind

Konstantinos Horaites, Stanislav Boldyrev

Physics Department, University of Wisconsin-Madison

Abstract

Although the temperature and density of solar wind electron velocity distribution functions (eVDFs) vary significantly as a function of heliocentric distance r, the shape of the distributions—characterized by a thermal core and suprathermal tails—varies only weakly. We suggest that this may be due to the peculiar conditions of the solar wind; specifically, the observed radial density and temperature profiles are such that the ratio between the mean free path λ_{mfp} and the characteristic distance L_T=T/|dT/dx| over which the temperature varies is nearly constant. If $\gamma \equiv \lambda_{mfp}/L_T$ is exactly constant, then the collisional kinetic equation admits self-similar solutions. We discuss these solutions and their applicability to the solar wind near 1 AU.

Introduction

In a plasma where γ=constant, the collisional kinetic equation admits self-similar solutions [1]. Consider the independent variables $\xi \equiv \mathrm{v}^2 = (V/V_{th})^2$ ($\mathbf{V}$ is velocity, $V_{th} = \sqrt{\frac{2T}{m}}$) and $\mu \equiv \cos\theta$, where θ is the angle between $\mathbf{V}$ and the x-axis. x is the 1D spatial coordinate of variation. We assume a self-similar form for the distribution function f(x, **v**, t):

$$f(x, \mathbf{v}, t) = \frac{NF(\mathbf{v}, t)}{T(x)^{\alpha}} \tag{1}$$

T is temperature, t is time, and N is a normalization factor. We impose $\int F d^3\mathrm{v} = 1$, and $\int f d^3V = n$, where n is the density. Assuming the electrons are gyrotropic and move in the proton frame, the kinetic equation can be written in the linearized form:

$$\begin{aligned}\frac{\partial F}{\partial t} = A\xi^{1/2}\Big[&- \gamma\mu\Big(\alpha F + \xi\frac{\partial F}{\partial \xi}\Big) \\ &+ \gamma\delta\Big(\mu\frac{\partial F}{\partial \xi} + \frac{(1-\mu^2)}{2\xi}\frac{\partial F}{\partial \mu}\Big) \\ &+ \frac{1}{\xi}\Big(\frac{\partial F}{\partial \xi} + \frac{\partial^2 F}{\partial \xi^2}\Big) \\ &+ \frac{1}{2\xi^2}\Big(- 2\mu\frac{\partial F}{\partial \mu} + (1-\mu^2)\frac{\partial^2 F}{\partial \mu^2}\Big)\Big]\end{aligned} \tag{2}$$

Where $A \equiv \frac{4\pi e^4\Lambda}{(2m)^{1/2}}$, and $\delta \equiv \left(\frac{eET}{2\pi e^4 \Lambda n}\right)/\gamma$.

Applicability

[1] set $\frac{\partial F}{\partial t} = 0$ and analyzed asymptotic regions of ξ theoretically. This theory appears to be appropriate for the solar wind for a number of reasons:

- Because $n \propto r^{-2}$, we require $T \propto r^{-1/2}$ to have γ =constant, very close to observations.
- Suprathermal power law tails $F \propto \xi^{-\alpha}$ are predicted $\Rightarrow$ halo or superhalo? [2]
- Heat flux as a function of γ follows Spitzer-Härm theory for $\gamma << 1$, then approaches a constant as $\gamma \to \infty$, agreeing with recent measurements [3].
- Theory calls for the presence of a large-scale electric field **E**, that ensures current balance.
- Self-similarity of eVDFs near 1 AU can be verified directly (see figure 1)
- Theory predicts that heat flux $q = \frac{m}{2}\int fV^3 \cos\theta d^3V$ scales like $q \propto r^{-11/4}$, very close to the typically measured scaling r^{-3}

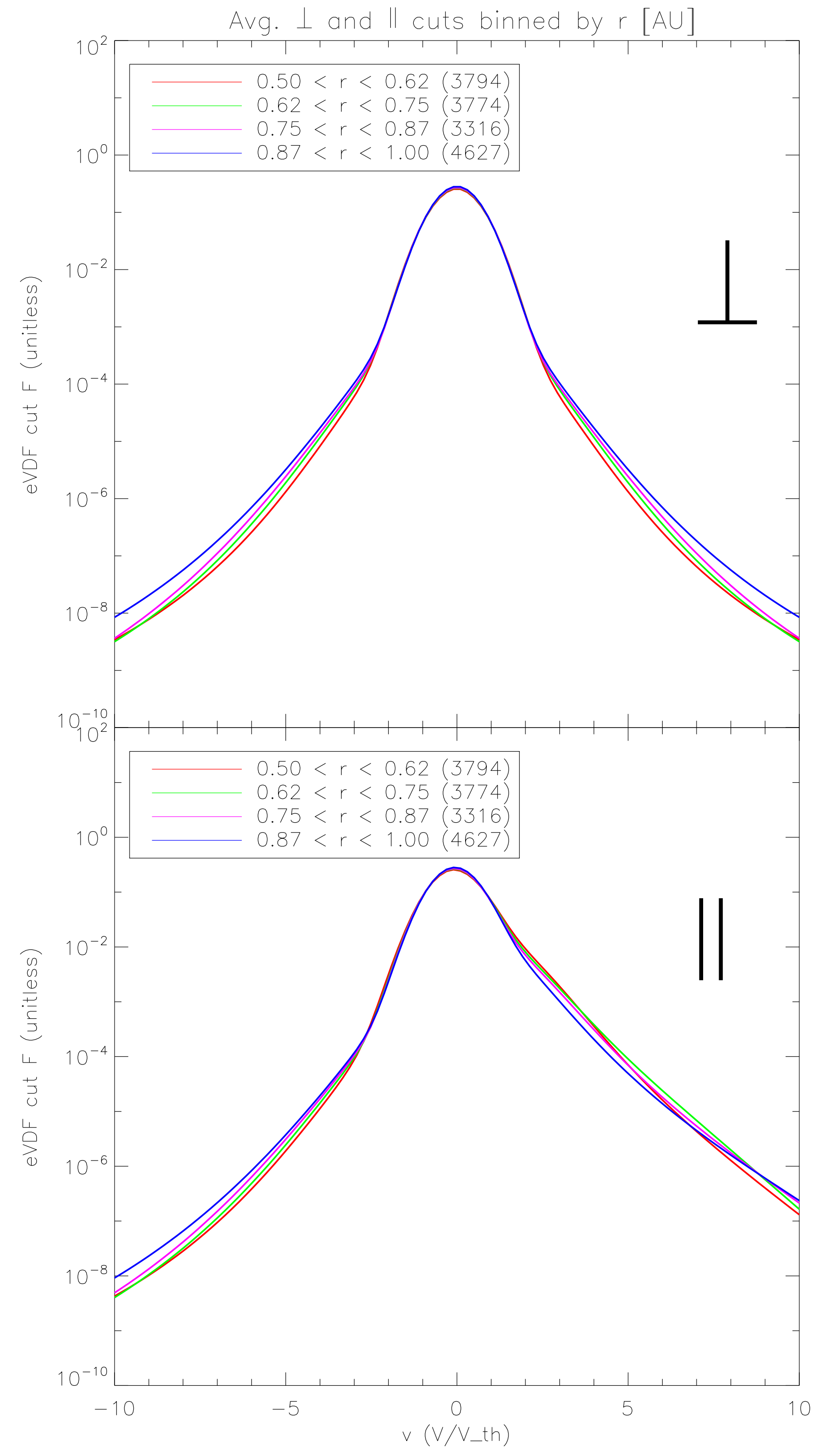

Figure 1: F variation with distance 0.5-1 AU from the Helios data. The gradual change of F with distance directly shows that the eVDFs are nearly self-similar.

Numerical Simulation

The steady state solution satisfies equation 2 with $\frac{\partial F}{\partial t} = 0$. To find this solution numerically, we use the *method of relaxation*. Starting with an initial guess for F, we simulate the evolution of F according to equation 2 using a finite difference scheme.

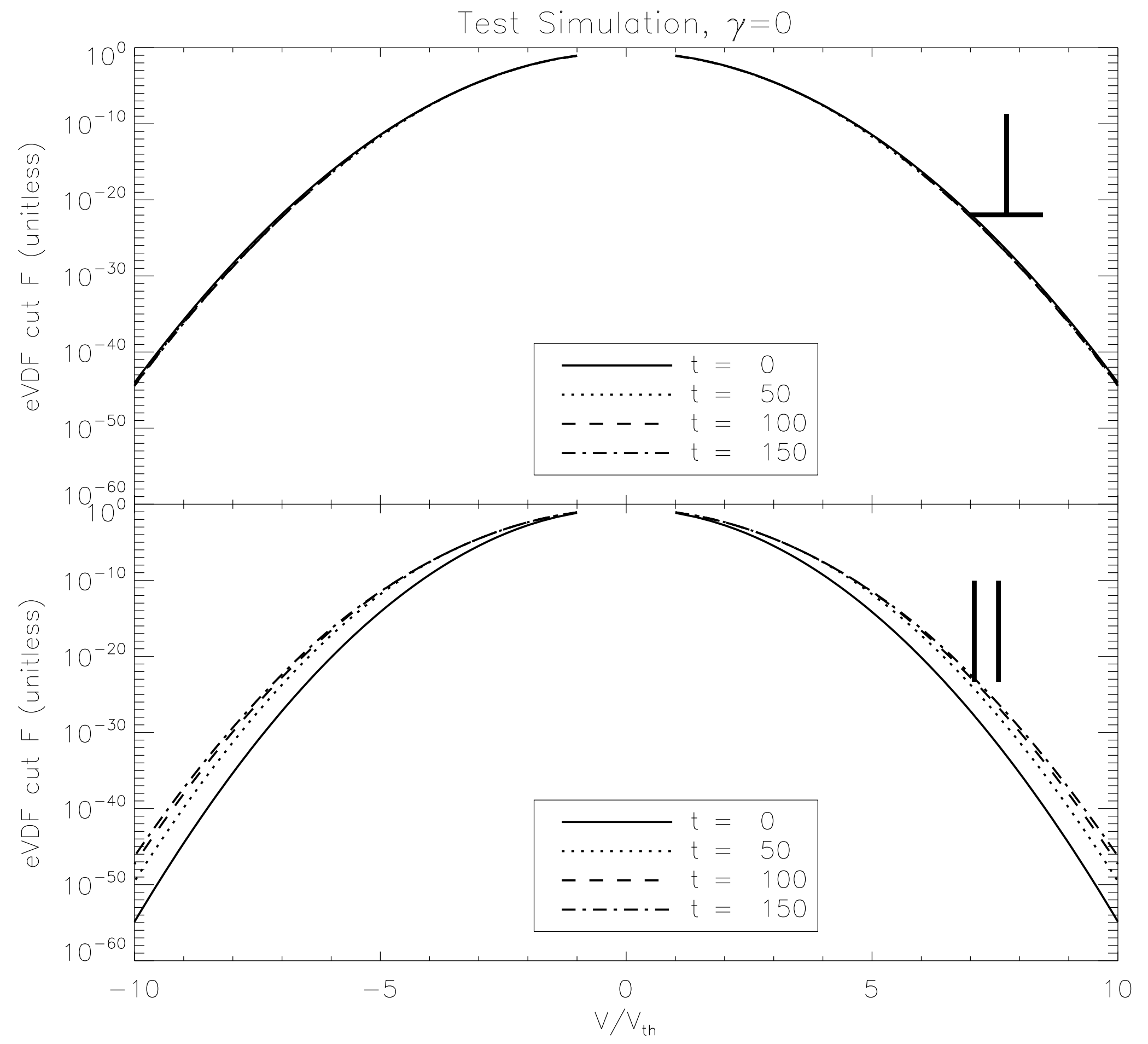

Figure 2: We test our code by setting $\gamma = 0$, $A = 1$. Here the effects of the electric field and temperature gradient are negligible. We initialize with an anisotropic bi-Maxwellian eVDF, and see that it converges towards an isotropic bi-Maxwellian.

If we allow γ to be non-zero, an electric field **E** will develop that counteracts the particle flux due to the temperature gradient.

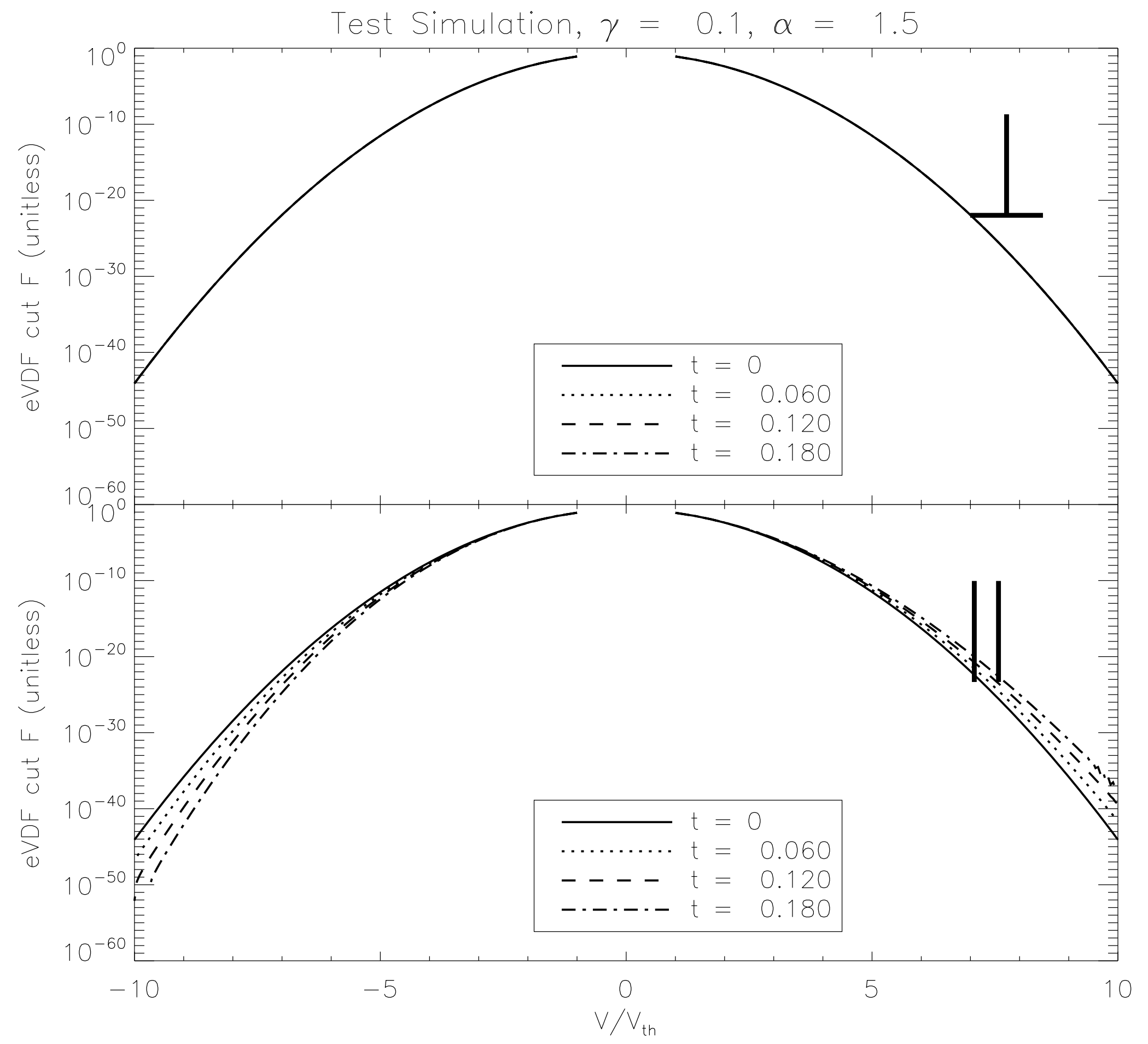

Figure 3: Set $\gamma = 0.1$, $A = 1$, $\alpha = 1.5$. Here the electric field and temperature gradient are important and a heat flux develops. We initialize as an isotropic bi-Maxwellian. The simulation shown has not yet converged.

Conclusion

The condition γ = *constant* is nearly satisfied in the solar wind, suggesting that eVDFs may be understood in terms of self-similar solutions of the kinetic equation. In reality γ might vary slowly with distance; however if the variation is sufficiently slow the same theory of self-similar solutions should be applicable. In this case we could use the local γ to find F everywhere.

The fine-tuning of the density and temperature profiles that leads to self-similarity merits the question: is this merely a coincidence? We speculate that the conditions in the solar wind may settle naturally into self-similarity, perhaps because of feedback between the steepness of the temperature gradient and the heat flux **q**, which is not divergenceless ($\nabla \cdot \mathbf{q} \neq 0$).

References

[1] S. I. Krasheninnikov and O. G. Bakunin.
Self-similar variables and the problem of nonlocal electron heat conductivity.
Technical report, October 1993.

[2] L. Wang, R. P. Lin, C. Salem, M. Pulupa, D. E. Larson, P. H. Yoon, and J. G. Luhmann.
Quiet-time Interplanetary ~2-20 keV Superhalo Electrons at Solar Minimum.
, 753:L23, July 2012.

[3] S. D. Bale, M. Pulupa, C. Salem, C. H. K. Chen, and E. Quataert.
Electron Heat Conduction in the Solar Wind: Transition from Spitzer-Härm to the Collisionless Limit.
, 769:L22, June 2013.

[4] I. F. Potapenko and S. I. Krasheninnikov.
Numerical solution of nonlinear electron kinetic equation in self-similar variables.
Journal of Plasma Physics, 77:803–812, December 2011.

Acknowledgements

This research was funded by US DoE award DE-SC0003888, DoE grant DE-SC0001794, and NSF grant PHY-0903872

Contact Information

- Group Website: http://home.physics.wisc.edu/~boldyrev/
- Email: horaites@wisc.edu