Application of Self-Similar Kinetic Theory to the Solar Wind

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Abstract

If the temperature Knudsen number $\gamma(x) = \lambda_{mfp}|\frac{dlnT}{dx}|$ in a plasma is constant thoughout the system, the collisional kinetic equation for electrons admits self-similar solutions. These solutions have the novel property that the "shape" of the eVDF does not vary in space. Such a theory should be applicable in the solar wind, where the density and temperature are observed to vary as power laws with heliocentric distance r such that $\gamma(r) \sim$constant. We present results of numerical simulations, where we find the steady-state eVDF for various γ. We then compare the predictions of the theory with satellite observations from the Helios and Wind missions. Overall, the theory exhibits remarkable consistency with a variety of electron measurements, and provides an intuitive context for understanding the steady-state solar wind eVDFs.

Introduction

Drift kinetic equation, ignoring $\mathbf{E}\times\mathbf{B}$ drifts:

$$\frac{\partial f}{\partial t}+V_\parallel\hat{b}\cdot\nabla f+\Big(\mu_B B\nabla\cdot\hat{b}+\frac{q_e E_\parallel}{m}\Big)\frac{\partial f}{\partial V_\parallel}=C(f) \quad (1)$$

If $\gamma = \frac{\lambda_{mfp}}{L_T}$ =constant, then for $v \equiv \frac{V}{V_{th}} >> 1$, eq. 1 reduces to equation 3, *independent of* x:

$$f \equiv \frac{NF(\mu,\xi,\tau)}{T(x)^\alpha}, \mu \equiv \cos\theta, \xi \equiv \Big(\frac{V}{V_{th}}\Big)^2, \tau = \nu t$$
$$\gamma \equiv \Big|\frac{T^2(dlnT/dx)}{2\pi e^4\Lambda n}\Big|, \gamma_E \equiv \frac{eET}{2\pi e^4\Lambda n}, \nu \equiv \frac{8\pi e^4\Lambda n}{m^2(V_{th})^3} \quad (2)$$

$$\frac{\partial F(\mu,\xi,\tau)}{\partial\tau} = \xi^{1/2}\Big\{-\gamma\Big[\alpha\mu F + \mu\xi\frac{\partial F}{\partial\xi}+\frac{\alpha_B}{2}(\alpha+1/2)(1-\mu^2)\frac{\partial F}{\partial\mu}\Big]+ \gamma_E\Big[\mu\frac{\partial F}{\partial\xi}+\frac{1-\mu^2}{2\xi}\frac{\partial F}{\partial\mu}\Big]+ \frac{1}{\xi}\Big[\frac{\partial F}{\partial\xi}+\frac{\partial^2 F}{\partial\xi^2}\Big]+\frac{\beta}{2\xi^2}\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial F}{\partial\mu}\Big\} \quad (3)$$

Applicability

Equation 3 assumes power law variation along $\hat{B}$: $n \propto x^{\alpha_n}$, $T \propto x^{\alpha_T}$, $B \propto x^{\alpha_B}$. The power law indices α_n and α_T are such that γ is nearly constant as a function of heliocentric distance in the solar wind.

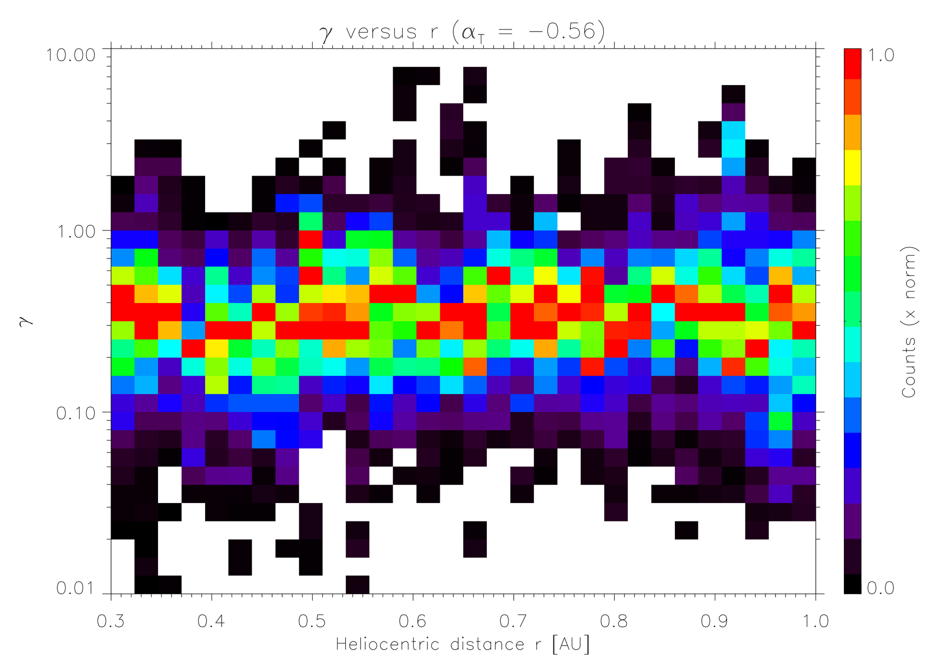

Figure 1: If $\gamma = constant$, equation 3 applies. Histogram of γ (columns normalized by peaks) 0.3-1 AU, Helios data.

Numerical Simulation

The steady state solution satisfies equation 3 with $\frac{\partial F}{\partial\tau} = 0$. To find this solution numerically, we use the *method of relaxation*. Starting with an initial guess for F, we simulate the evolution of F according to equation 3 using stochastic *Langevin equations*, until a steady state is reached.

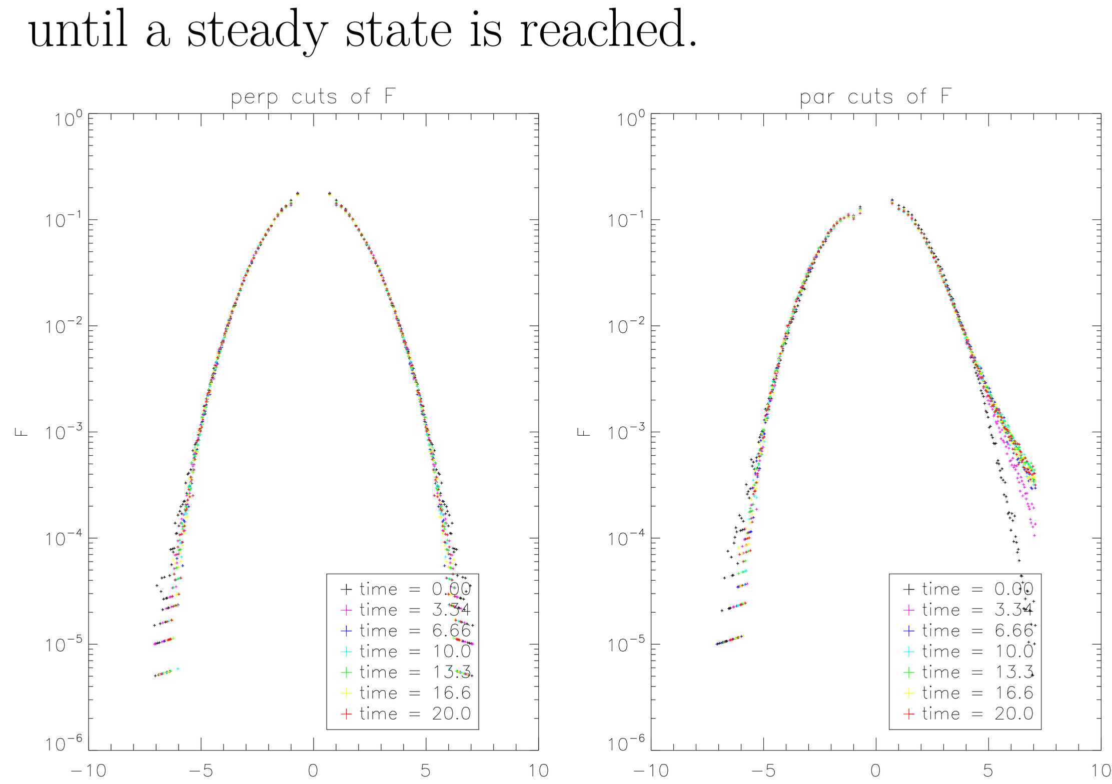

Figure 2: Time evolution of $\perp$ and $\parallel$ cuts of F, for $\gamma = 0.05$. The simulation approaches a steady state.

Simulation/Data Comparison

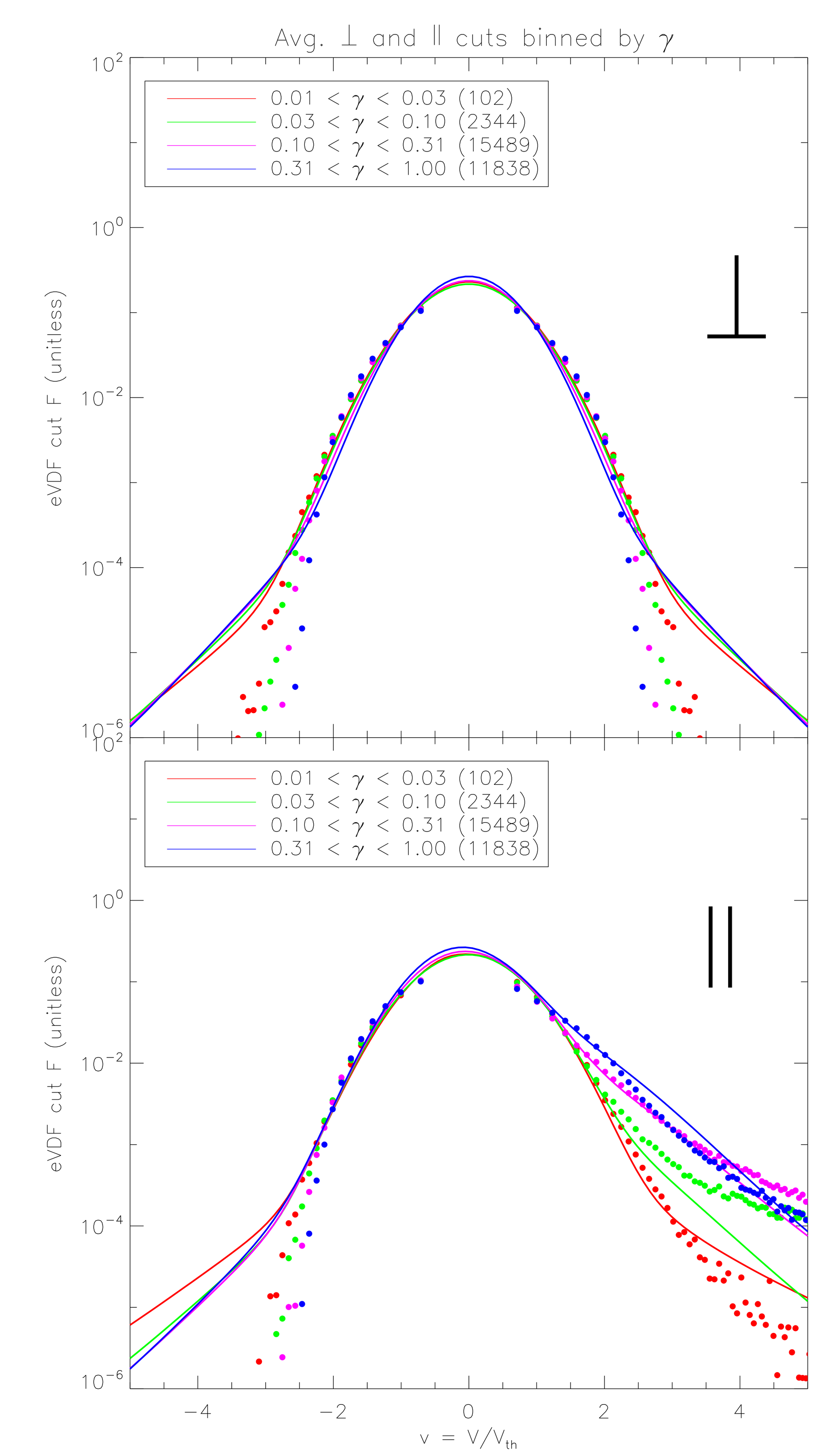

Figure 3: Results from Langevin simulations (points) plotted over average eVDF cuts of Helios data (lines), for comparable γ

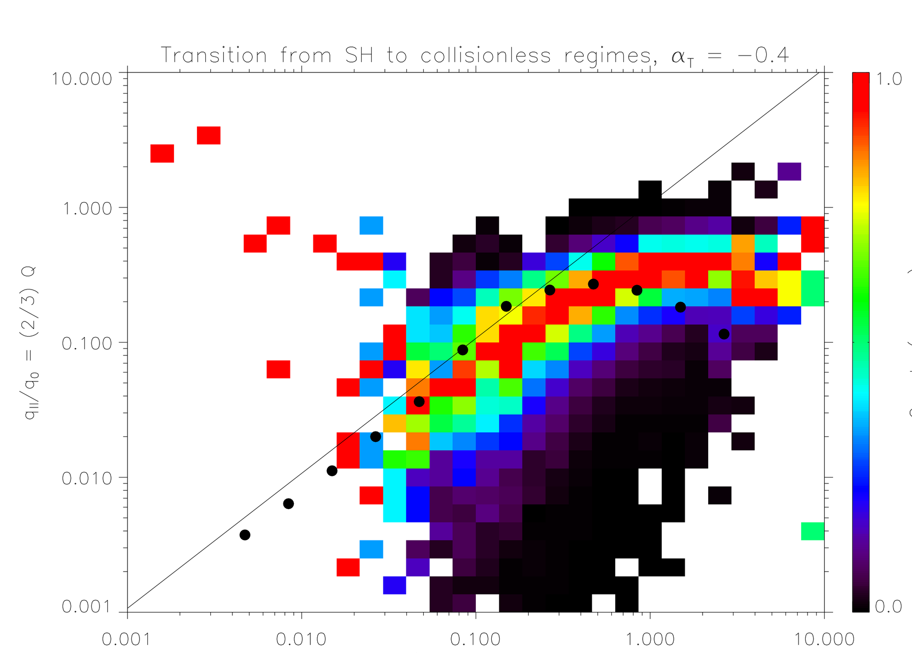

Figure 4: Comparison of heat flux obtained from Helios data (2D histogram with columns normalized by peaks), and Langevin simulations (points). A break is observed between the Spitzer-Härm ($\gamma << 1$) and collisionless ($\gamma \gtrsim 1$) limits at $\gamma \approx 0.3$

Conclusions

- In the solar wind $\gamma \approx$constant, allowing self-similar kinetic equation to be applied.
- Average cuts of Helios data match the results of simulations for the core and strahl populations, but not for the isotropic halo.
- Transition from Spitzer-Härm to collisionless regimes is correctly predicted.

References

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