Application of Self-Similar Kinetic Theory to the Solar Wind

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Abstract

If the temperature Knudsen number $\gamma(x) \sim \frac{T(dT/dx)}{n}$ in a plasma is constant thoughout the system, the collisional kinetic equation for electrons admits self-similar solutions. These solutions have the novel property that the "shape" of the electron velocity distribution function (eVDF) does not vary in space. Such a theory should be applicable to the solar wind in the inner heliosphere, where the density and temperature are observed to vary as power laws with heliocentric distance r such that $\gamma(r) \sim$ constant. We present results of numerical simulations, where we find the steady-state eVDF for various γ. We then compare our predictions with observations from the Helios satellite. Our theory successfully produces a strahl population, which we interpret to be comprised of thermal runaway electrons that originated from the corona. For the large (collisionless) Knudsen numbers that are typically observed in the solar wind, this population contributes significantly to the total electron heat flux.

Introduction

$$f \equiv \frac{NF(\mu,\xi,\tau)}{T(x)^{\alpha}}, \mu \equiv \cos\theta, \xi \equiv \left(\frac{V}{V_{th}}\right)^2$$
$$\gamma \equiv -\frac{T^2(d\ln T/dx)}{2\pi e^4 \Lambda n}, \gamma_E \equiv \frac{eET}{2\pi e^4 \Lambda n}, B \sim x^{\alpha_B} \qquad (1)$$

Drift kinetic equation, ignoring $\mathbf{E}\times\mathbf{B}$ drifts:

$$\frac{\partial f}{\partial t} + V_{\parallel}\hat{b}\cdot\nabla f + \left(\mu_B B \nabla\cdot\hat{b} + \frac{q_e E_{\parallel}}{m}\right)\frac{\partial f}{\partial V_{\parallel}} = C(f) \qquad (2)$$

If $\gamma(x) \sim \frac{T(dT/dx)}{n} =$ const., and $n(x), T(x), B(x)$ vary as power laws, then for $\xi \equiv \left(\frac{V}{V_{th}}\right)^2 >> 1$, eq. 2 reduces to equation 3, *independent of* x:

$$\frac{\partial F(\mu,\xi,\tau)}{\partial \tau} = \xi^{1/2}\Big\{\gamma\Big[-\alpha\mu F - \mu\xi\frac{\partial F}{\partial \xi} + \frac{-\alpha_B}{2}(\alpha+1/2)(1-\mu^2)\frac{\partial F}{\partial \mu}\Big] + \gamma_E\Big[\mu\frac{\partial F}{\partial \xi} + \frac{1-\mu^2}{2\xi}\frac{\partial F}{\partial \mu}\Big] + \frac{1}{\xi}\Big[\frac{\partial F}{\partial \xi} + \frac{\partial^2 F}{\partial \xi^2}\Big] + \frac{\beta}{2\xi^2}\frac{\partial}{\partial \mu}(1-\mu^2)\frac{\partial F}{\partial \mu}\Big\} \qquad (3)$$

Applicability

Equation 3 assumes power law variation along $\hat{B}$: $n \propto x^{\alpha_n}$, $T \propto x^{\alpha_T}$, $B \propto x^{\alpha_B}$. The power law indices α_n and α_T are such that γ is nearly constant as a function of heliocentric distance in the solar wind.

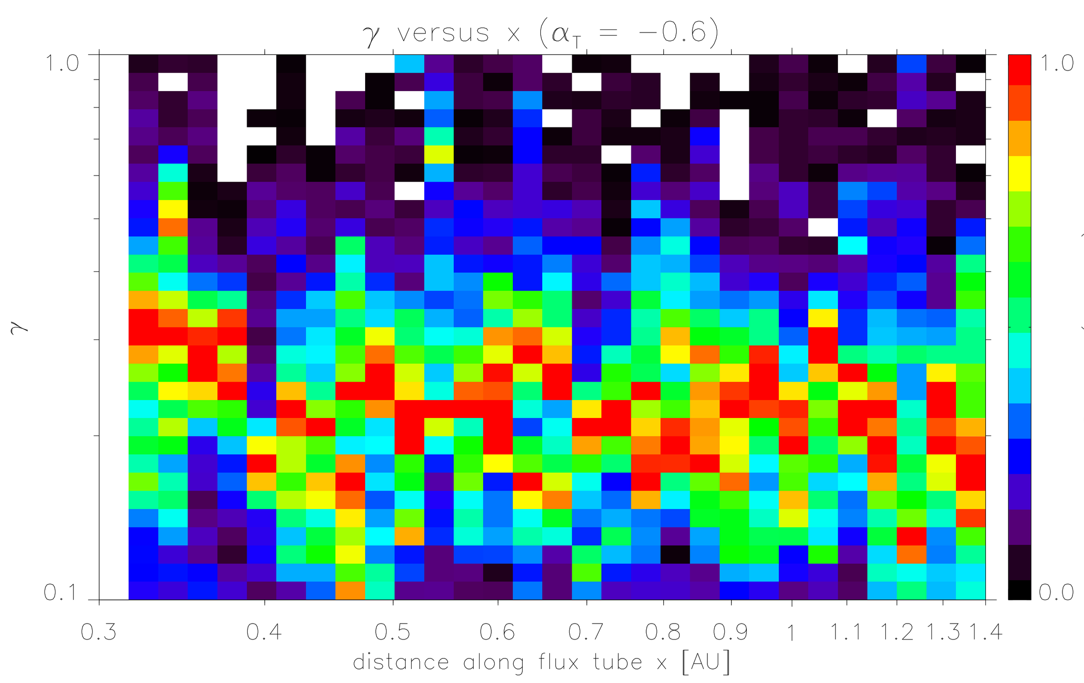

Figure: If γ =const., equation 3 applies. Histogram of γ (columns normalized by peaks) 0.3-1 AU, Helios data.

Numerical Simulation

The steady state solution satisfies equation 3 with $\frac{\partial F}{\partial \tau} = 0$. To find this solution numerically, we use the *method of relaxation*. Starting with an initial guess for F, we simulate the evolution of F according to equation 3 using stochastic *Langevin equations*, until a steady state is reached.

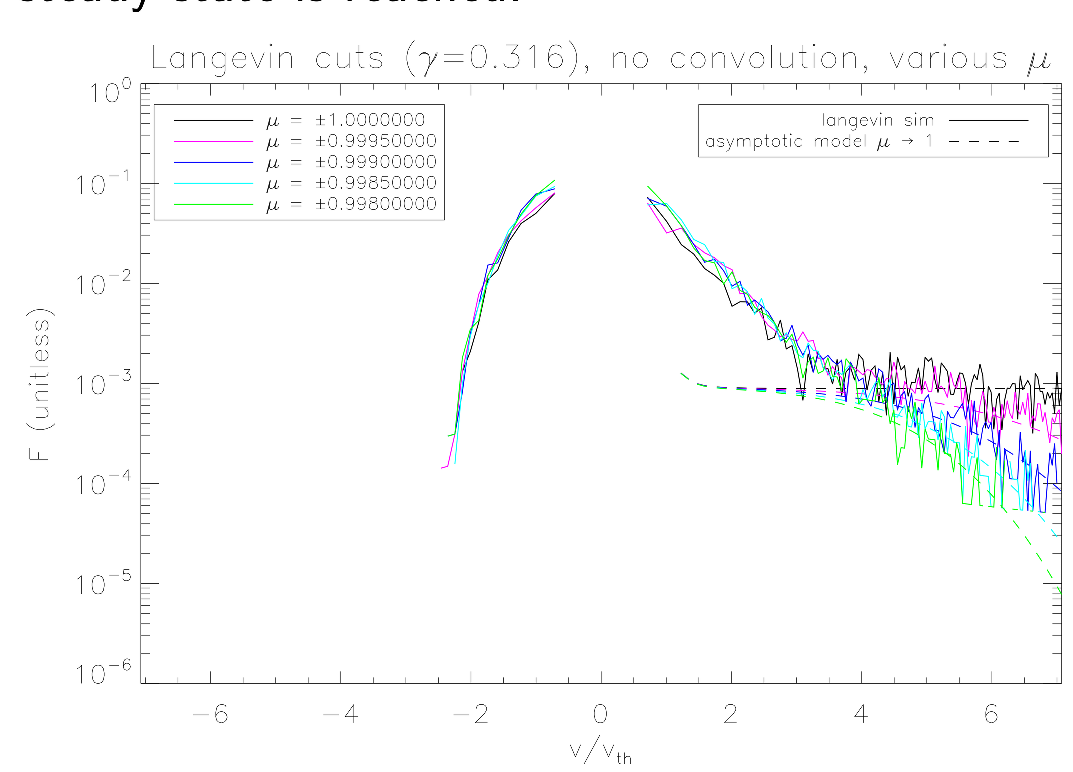

Figure: Cuts of the distribution along various μ. In the asymptotic regime $\mu \to 1$, $\xi >> 1$, the self-similar kinetic equation can be solved for a particular solution (equation 4). Our simulations match this asymptotic solution, which implies the formation of a very narrow ($< 1^{\circ}$ wide) strahl population.

Asymptotic Solution

In the asymptotic regime $\mu \to 1$, $\xi >> 1$, the self-similar kinetic equation has particular solutions of the form:

$$F(\xi,\mu) \sim C\xi^{\alpha'-\alpha}\exp\left\{\frac{\gamma\alpha'\xi^2(1-\mu)}{\beta}\right\} \qquad (4)$$

Simulation/Data Comparison

Because the E1 detector has a broad field of view, it tends to smear out narrow features in the distribution function such as the strahl; we denote the resulting convoluted (self-similar) function as $F^*(\mathbf{v}/v_{th})$. As described in [1], we apply a convolution to our numerical solutions of the kinetic equation, and compare this with the measured F^*.

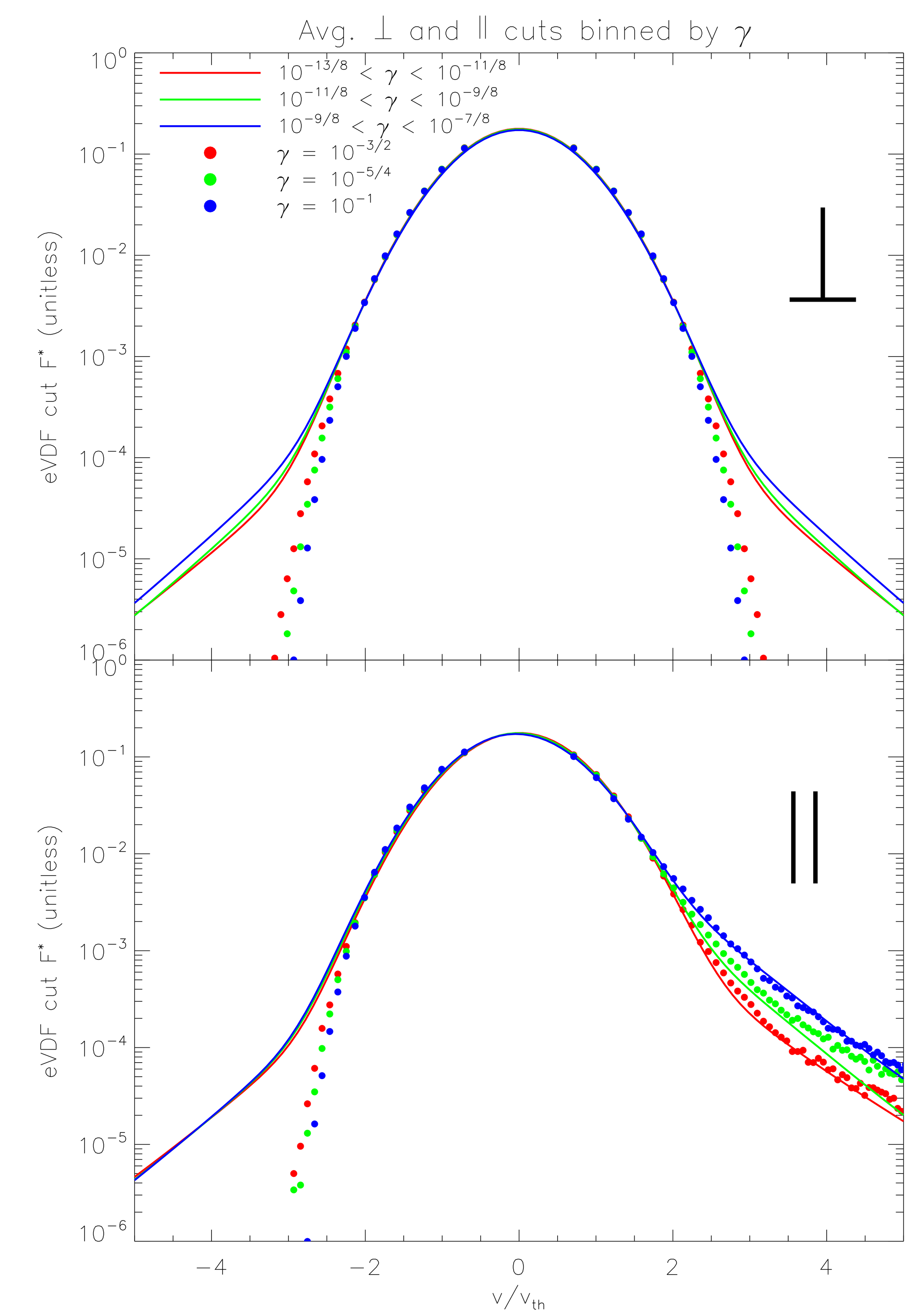

Figure: Results from Langevin simulations (points) plotted over average eVDF cuts of Helios data (lines), for comparable γ

Simulation/Data Comparison

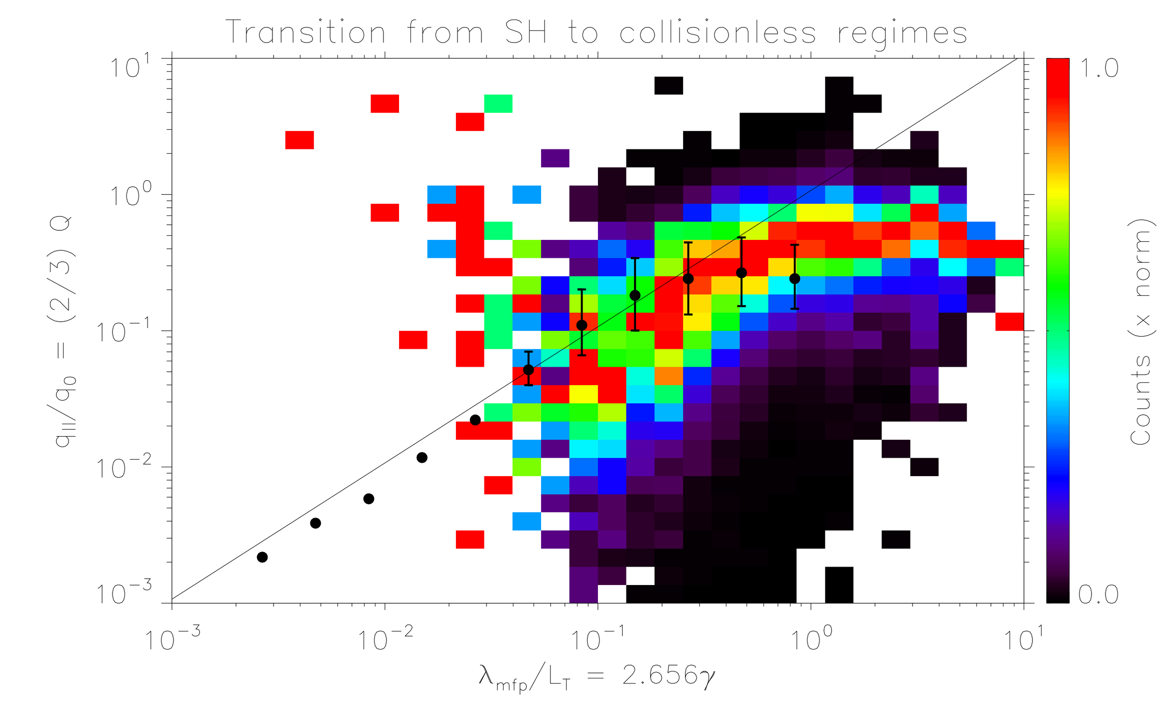

Figure: Comparison of heat flux obtained from Helios data (2D histogram with columns normalized by peaks), and Langevin simulations (points). A break is observed between the Spitzer-Härm ($\gamma << 1$) and collisionless ($\gamma \gtrsim 1$) limits at $\gamma \approx 0.3$

Conclusions

- In the solar wind $\gamma \approx$const., allowing self-similar kinetic equation to be applied.
- Average cuts of Helios data match the results of simulations for the core and strahl populations, but not for the isotropic halo.
- Transition from Spitzer-Härm to collisionless regimes is correctly predicted.

References

[1] K. Horaites, S. Boldyrev, S. I. Krasheninnikov, C. Salem, S. D. Bale, and M. Pulupa.
Self-similar theory of thermal conduction and application to the solar wind.
Phys. Rev. Lett., 114:245003, Jun 2015.

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