
Self-Similar Distribution Functions in the Solar Wind

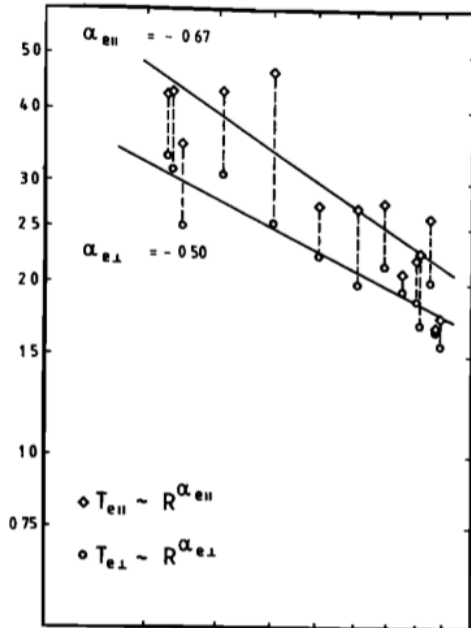
Konstantinos Horaites*

Stanislav Boldyrev*

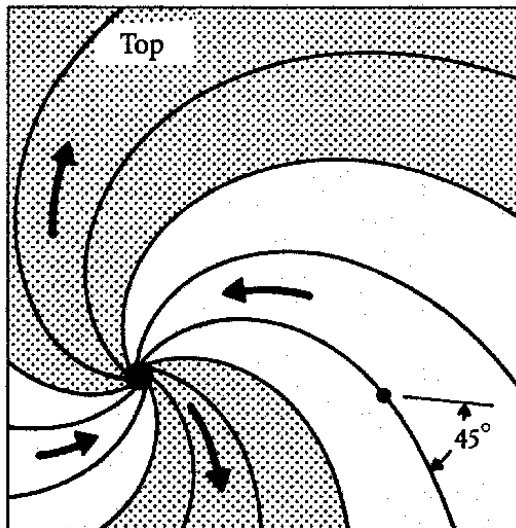
**University of Wisconsin-Madison*

Background: Solar Wind Electrons

LOW SPEED
STREAMS
(L)

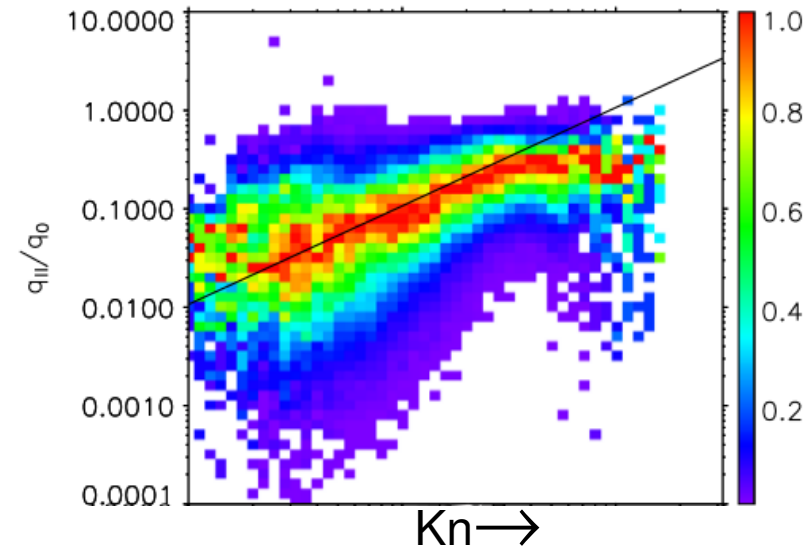
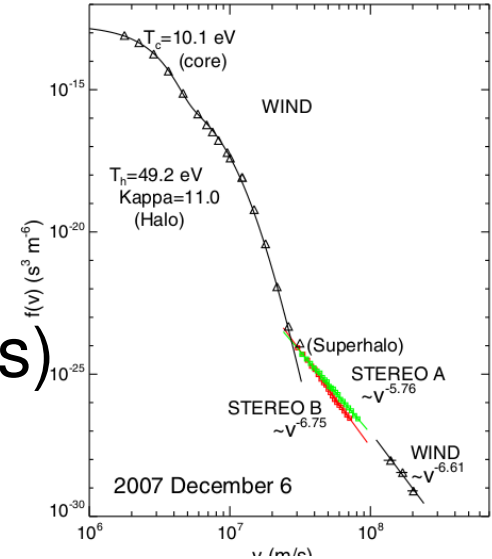


- Power laws:
 $n(r) \sim r^{-2}$
 $T(r) \sim r^{-1/2}$
 $q(r) \sim r^{-3}$
 (Pilipp 1990)



- Parker Spiral

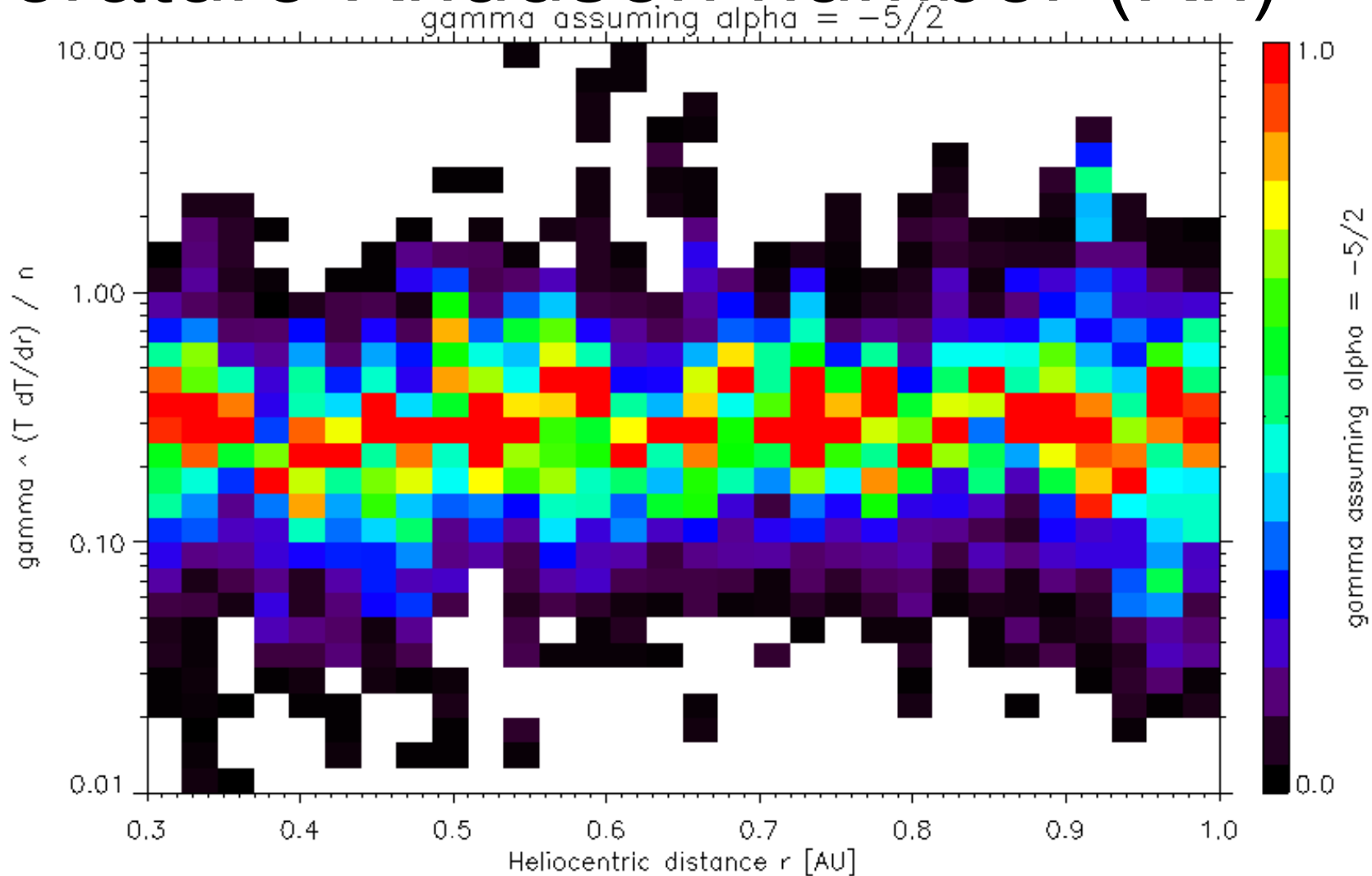
- Non-thermal electron velocity distribution functions (eVDFs) (Wang 2012)



- Spitzer Conductivity \rightarrow free streaming (Bale 2013)

Temperature Knudsen number (Kn)

Helios
Data



$Kn \sim (d \ln T / dr) * L_{\text{mfp}}$ | $Kn \rightarrow 0$: highly collisional
 Constant on average! | $Kn \sim 1$ weakly collisional

Self-Similar eVDFs (Krasheninnikov 1988)

$$\begin{aligned} \frac{\partial F}{\partial t} = & A\xi^{1/2} \left[-\gamma\mu \left(\alpha F + \xi \frac{\partial F}{\partial \xi} \right) \right. && \leftarrow \text{Advection} \\ & + \delta \left(\mu \frac{\partial F}{\partial \xi} + \frac{(1-\mu^2)}{2\xi} \frac{\partial F}{\partial \mu} \right) && \leftarrow \text{E Field} \\ & + \frac{1}{\xi} \left(\frac{\partial F}{\partial \xi} + \frac{\partial^2 F}{\partial \xi^2} \right) && \leftarrow \text{e-e energy exchange} \\ & \left. + \frac{1}{2\xi^2} \frac{\partial}{\partial \mu} (1-\mu^2) \frac{\partial F}{\partial \mu} \right] && \leftarrow \text{pitch angle scattering} \end{aligned}$$

If the Knudsen number ($\sim\gamma$) = constant,
then the kinetic equation can be written in a
spatially independent form!

Account for magnetic field geometry!

$$\frac{\partial f_0}{\partial t} + (\mathbf{U}_d + v_{\parallel} \mathbf{b}) \cdot \nabla f_0 - \left(\mathbf{b} \cdot \frac{D\mathbf{U}_d}{Dt} - \mu B \nabla \cdot \mathbf{b} - \frac{e}{m} E_{\parallel} \right) \frac{\partial f_0}{\partial v_{\parallel}} = 0$$



Diverging magnetic field
direction leads to collimation

Extend theory by starting from drift kinetic
equation, which accounts for a spatially varying
magnetic field

Applicable to solar wind? **YES**

- Non-thermal eVDFs



- Power laws: $n(r) \sim r^{-2}$

$$T(r) \sim r^{-1/2}$$

$$Kn \sim (T \, dT/dr)/n$$

—————→ $Kn = \text{constant!}$

Predicts $q(r) \sim r^{-11/4}$ (nearly r^{-3})



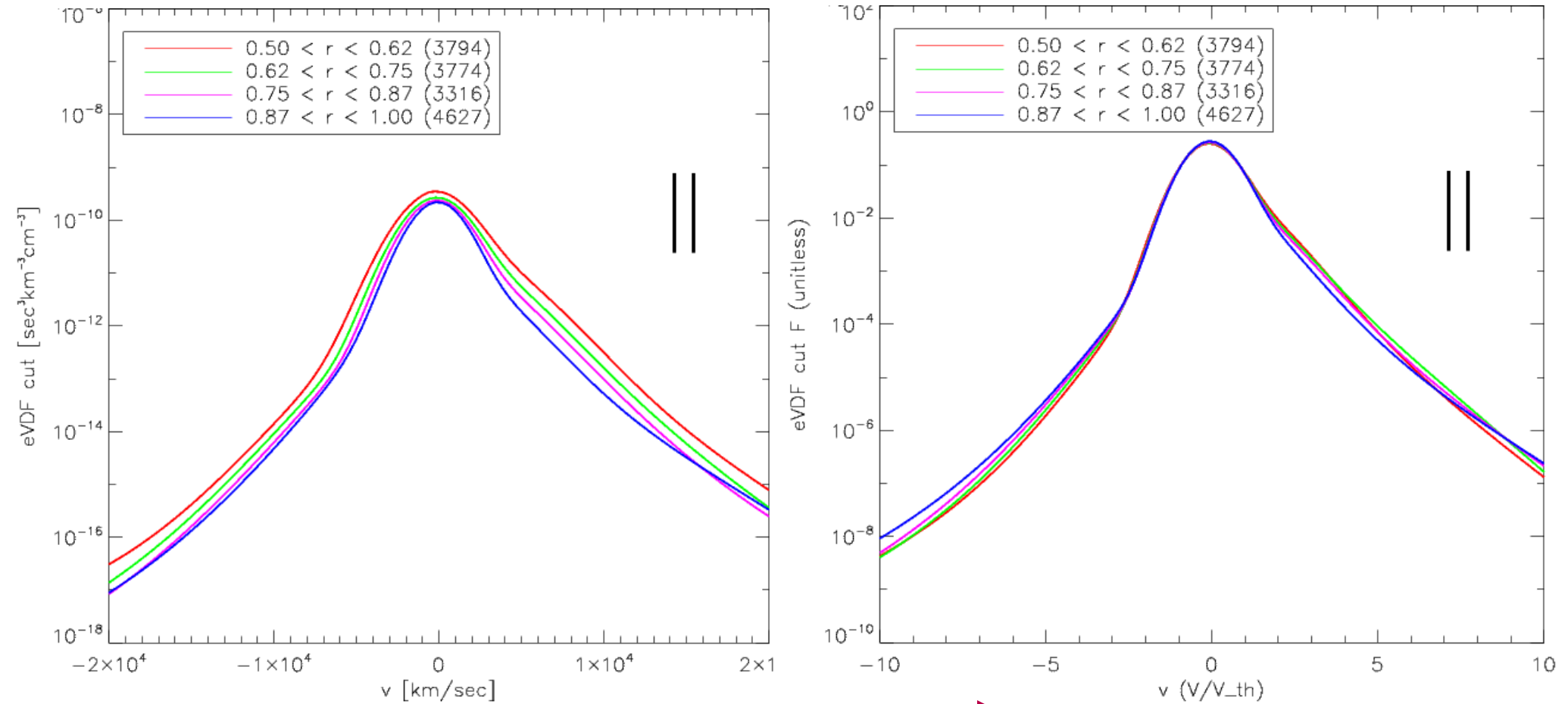
- Spitzer Conductivity → free streaming

- Parker Spiral



Also Verify Self-Similarity Directly

(Averaged Helios 1 eVDFs: 0.5-1 AU)



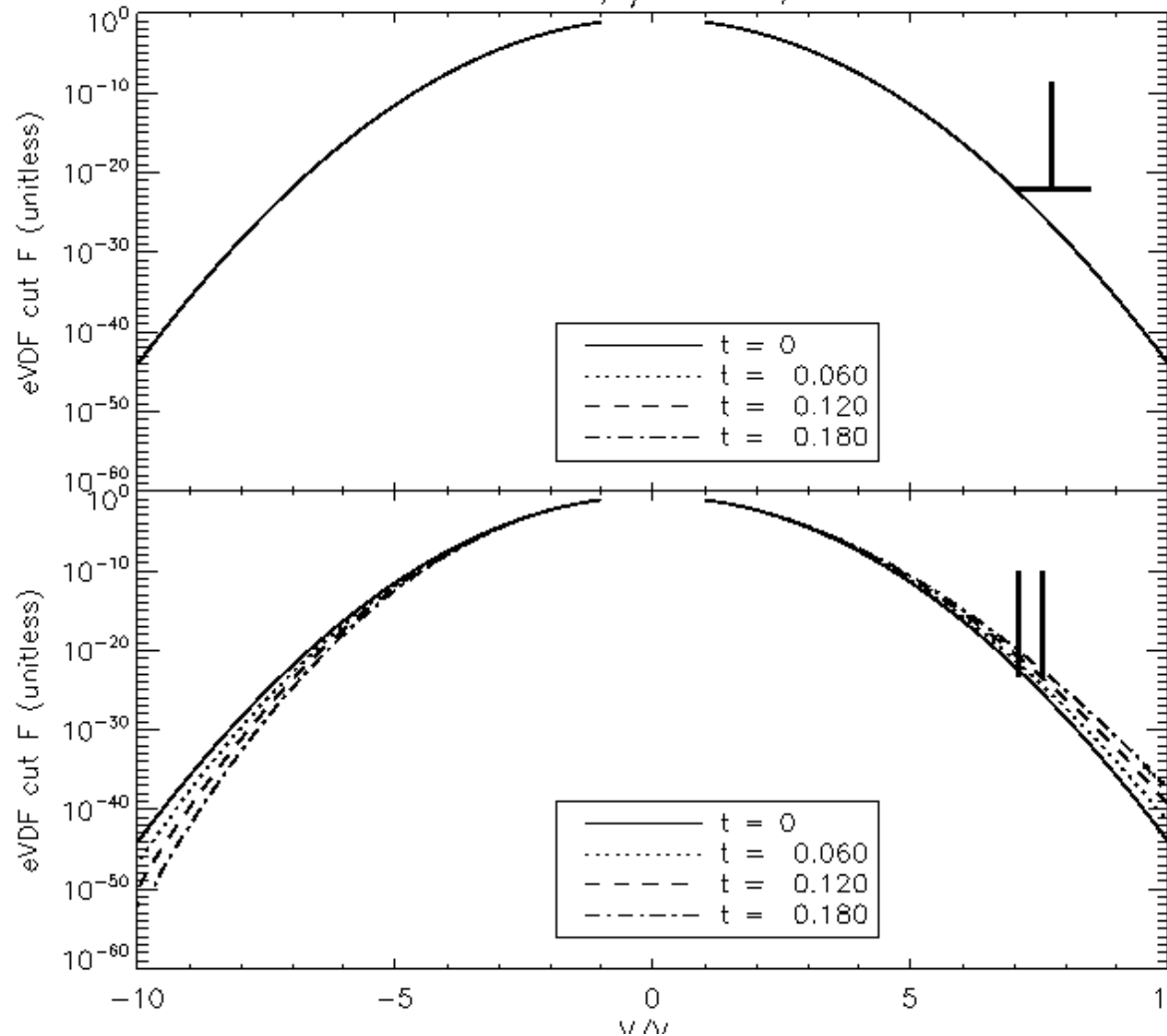
$f(x, v)$

Normalize

$F(x, v/v_{th}) \sim F_0(v/v_{th})$

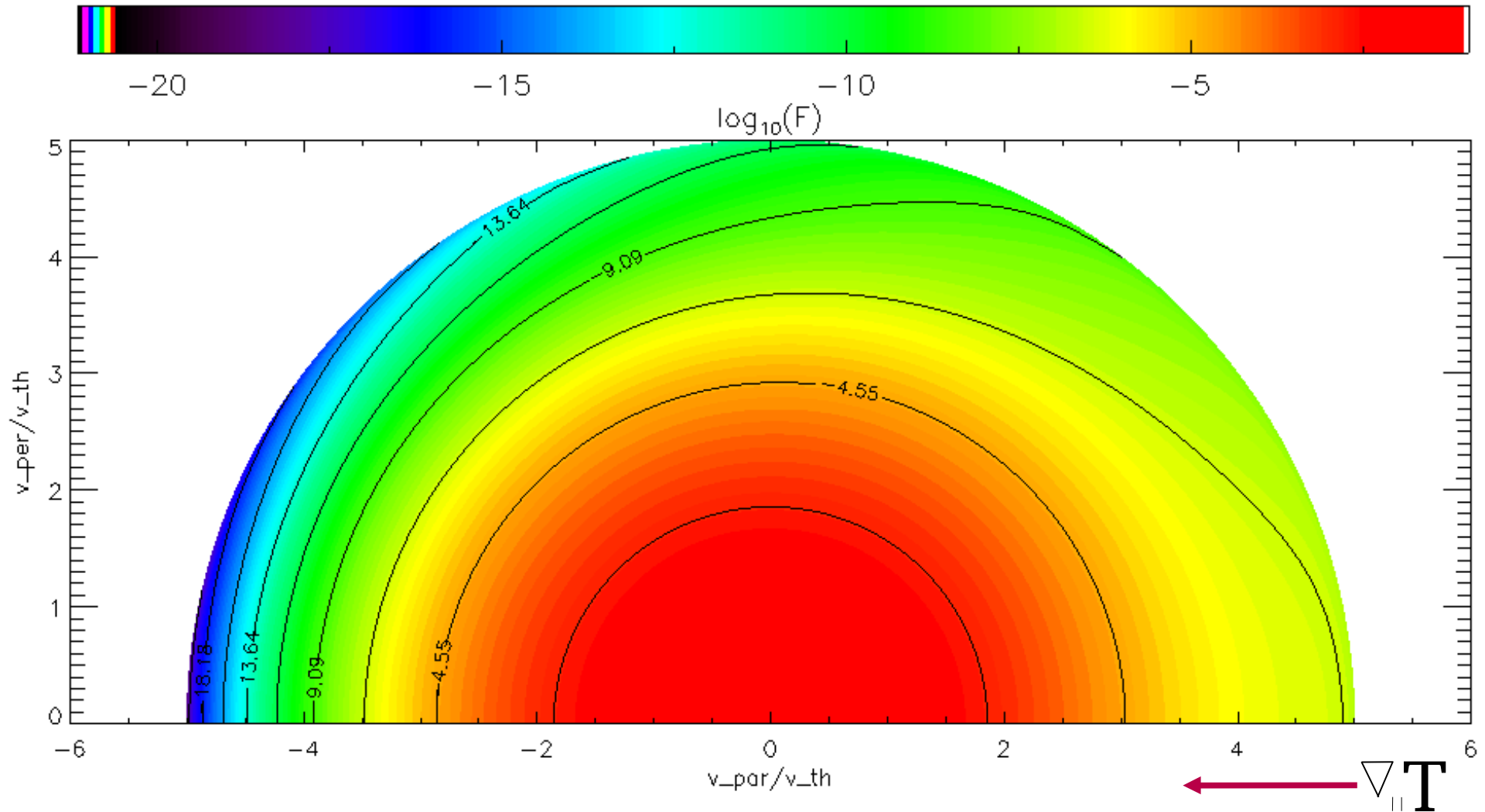
Simulations

Test Simulation, $\gamma = 0.1$, $\alpha = 1.5$



Test case with no magnetic field. Heat flux (from skewness of distribution) develops in time

Simulations



γ (Kn) = 0.004

Heat flux and (slight) collimation

Conclusions

Knudsen number in solar wind is constant as a function of radial distance

Theory of self-similar kinetic equation is consistent with measured properties of solar wind. Measured eVDF profiles indicate self-similarity

Hope to explain solar wind eVDFs with simulations