Application of Self-Similar Kinetic Theory to the Solar Wind: Data and Simulations

Konstantinos Horaites[1], Stanislav Boldyrev[1], S. Krasheninnikov[2], Chadi Salem[3], Stuart Bale[3,4], Marc Pulupa[3]
[1]Physics Department, University of Wisconsin-Madison,
[2]Mechanical and Aerospace Engineering Department, University of California-San Diego,
[3]Space Sciences Laboratory, University of California-Berkeley,
[4]Physics Department, University of California-Berkeley

APS-DPP Meeting, October 29

Theory: Background

Drift Kinetic Equation (ignore $\mathbf{E} \times \mathbf{B}$ drifts):

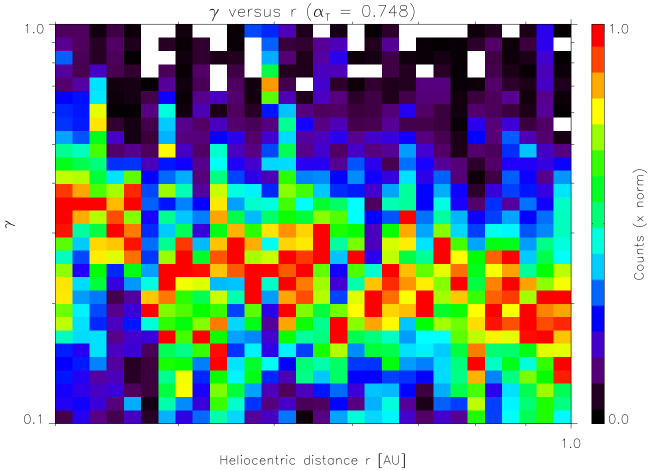
$$\frac{\partial f}{\partial t} + \mathsf{V}_\parallel \hat{b} \cdot \nabla f + \left(\mu_B B \nabla \cdot \hat{b} + \frac{q_e E_\parallel}{m}\right) \frac{\partial f}{\partial \mathsf{V}_\parallel} = C(f) \qquad (1)$$

If Knudsen number (usually denoted Kn) $\gamma \sim \frac{\lambda_{mfp}}{L_T}$ =constant, then for $\mathsf{v} \equiv \frac{\mathsf{V}}{\mathsf{V}_{th}} >> 1$, reduces to an equation *independent of* $\mathbf{x}$

$$f(\mathbf{x}, \mathbf{V}, t) \equiv \frac{NF(\mu, \xi, t)}{T(\mathbf{x})^\alpha}, \mu \equiv \cos\theta, \xi \equiv \left(\frac{\mathsf{V}}{\mathsf{V}_{th}}\right)^2 \qquad (2)$$

$$\frac{\partial F(\mu,\xi,t)}{\partial t} = \nu \xi^{1/2} \Big\{ \gamma \Big[-\alpha\mu F - \mu\xi \frac{\partial F}{\partial \xi} + \frac{-\alpha_B}{2}(\alpha + 1/2)(1-\mu^2)\frac{\partial F}{\partial \mu}\Big] + \gamma_E \Big[\mu \frac{\partial F}{\partial \xi} + \frac{1-\mu^2}{2\xi}\frac{\partial F}{\partial \mu}\Big] + \frac{1}{\xi}\Big[\frac{\partial F}{\partial \xi} + \frac{\partial^2 F}{\partial \xi^2}\Big] + \frac{\beta}{2\xi^2}\frac{\partial}{\partial \mu}(1-\mu^2)\frac{\partial F}{\partial \mu} \Big\} \qquad (3)$$

Applicability: $\gamma = \frac{\lambda_{mfp}}{L_T}$ =constant?



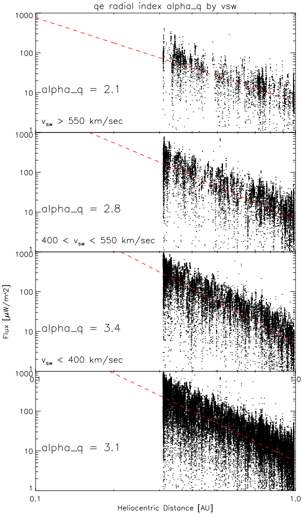
$\gamma \propto \frac{T(dT/dr)}{n}$ plotted versus heliocentric distance $0.3 < r < 1$ AU. (Helios electron data)

Applicability: Power Laws $X \propto r^{\alpha_X}$

n, T, q, B go as power laws in solar wind. Choose α_n and α_T, α_q are specified.

-	α_{model}	$\alpha_{observed}$
n	α_n	-2.2
T	$(\alpha_n + 1)/2$	-0.7
q	$(5\alpha_n - 1)/4$	-3.1
B	any	-1.6

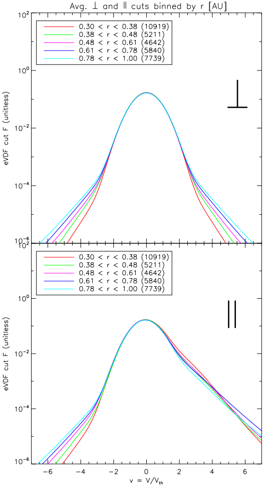
Theory matches well!
"Observed" values taken from fits to Helios data $0.3 < r < 1$ AU.



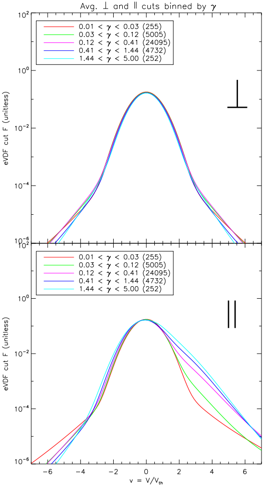
Note: power law index may depend on wind speed! Above: measurement of α_q

Applicability: Helios fits

Normalize Helios fits by $F \equiv \frac{f(\mathbf{x},\mathbf{V})T(\mathbf{x})^{\alpha}}{N}$ to get self-similar distribution function F. Self-similarity realized for core and strahl.



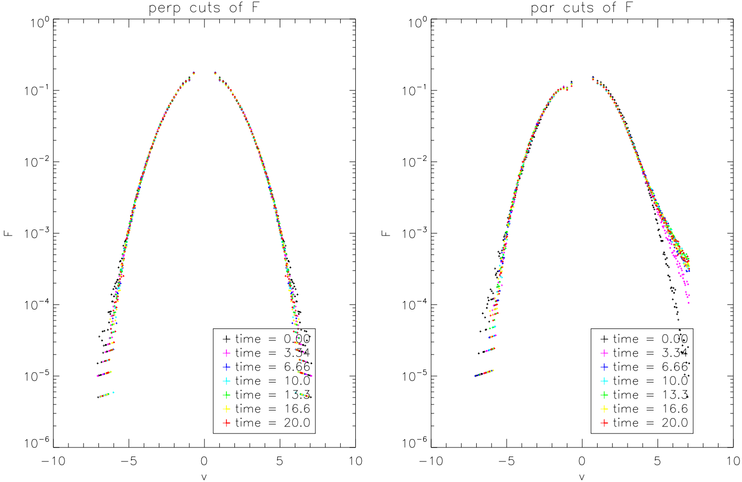
$0.3 < r < 1$ AU



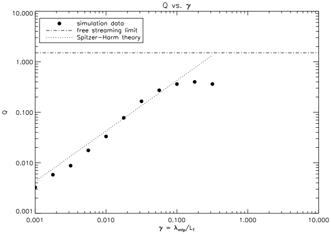
$0.01 < \gamma < 5$

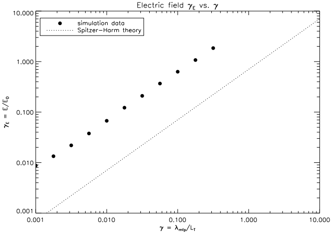
Langevin Simulations

Simulate time-dependent kinetic equation, by deriving stochastic *Langevin equations*. Populate phase space (μ, ξ) with N_p particles, and as $N_p \to \infty$, exact solution is obtained. Below: cuts versus time, $\gamma = 0.05$, $N_p = 1e7$, $\alpha_n = -1.8$, $\alpha_T = -0.4$, $\alpha_B = -2$.



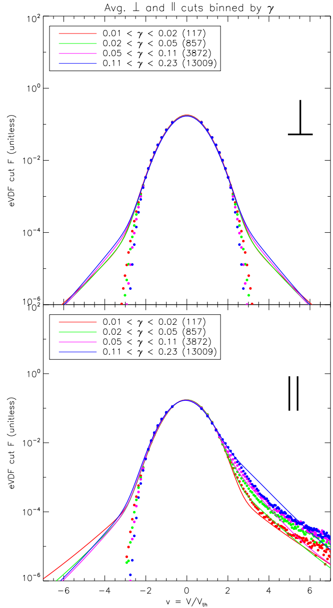
Comparison with Spitzer theory





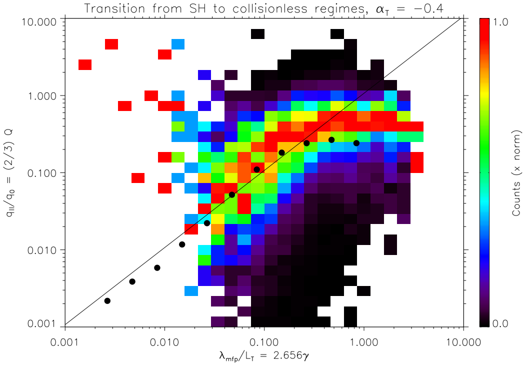
- $Q \equiv \int F \mathsf{v}_{\parallel} \mathsf{v}^2 d^3\mathsf{v}$
- Follows Spitzer-Härm relation $Q_{SH} \propto \gamma$ for $\gamma << 1$
- Transitions to collisionless heat flux at $\gamma \approx 0.1$
- Magnitude of Q depends on choice of v_{max}
- Electric field follows Spitzer-Härm scaling $\frac{E}{E_D} \propto \gamma$ in both regimes
- Can simulations be made to match theory exactly?

eVDF Cuts



- Comparison of simulations (points) with Helios eVDF cuts (lines) averaged into bins ordered by γ
- γ are logarithmically spaced
- High level of agreement in the core and strahl!
- Less agreement in the halo... not enough points in simulation?
- Convolute results of Langevin simulation with a function that represents the response of the detector, which measures over finite angle

Transition from Spitzer-Härm to Collisionless limit



- Histogram of $\frac{q_{\parallel}}{q_0}$, where $q_0 \equiv \frac{3}{2} n \mathrm{V}_{th} T$, vs. γ (see Bale, 2013)
- Langevin simulations (dots) match the data well
- Departure from expected form for $\gamma > 1$, probably because our collision operator doesn't apply for strongly non-Maxwellian core

Conclusions

- In the solar wind $\gamma \approx$constant, allowing self-similar kinetic equation to be applied
- Can order eVDF profiles by γ. Average Helios cuts match the results of simulations for core and strahl electron populations, but not for the halo population.
- Transition from Spitzer to collisionless regimes is predicted. However, trouble running simulations for $\gamma \gtrsim 1$.