Application of Self-Similar Kinetic Theory to the Solar Wind: Data and Simulations

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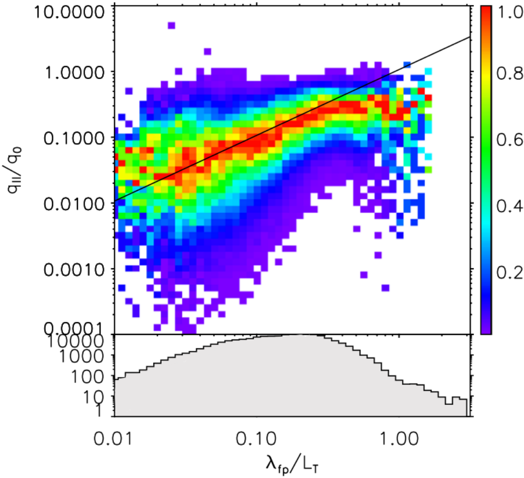
Thermal conductivity in the solar wind

$$\gamma = \lambda_{mfp}\frac{d\ln T}{dx} \propto \frac{T dT/dx}{n}$$

- $\gamma << 1$: $\mathbf{q} = -\kappa\nabla T$ (collisional)
- $\gamma >> 1 : q \sim nTv_{th}$ (collisionless)
- $0.01 \lesssim \gamma \lesssim 1 : \mathbf{q} = ???$ (weakly collisional)



Wind data, r=1 AU (Bale, 2013)

Theory: Background

Drift Kinetic Equation ($|\vec{V}| >> \mathrm{V}_{sw}$):

$$\frac{\partial f}{\partial t} + \mathrm{V}_{\parallel}\hat{b}\cdot\nabla f + \left(\mu_B B\nabla\cdot\hat{b} + \frac{q_e E_{\parallel}}{m}\right)\frac{\partial f}{\partial \mathrm{V}_{\parallel}} = \hat{C}(f)$$

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If $\gamma(x) =$constant, and $n(x)$, $T(x)$, $B(x)$ vary as power laws, then for $\frac{\mathrm{V}}{\mathrm{V}_{th}} >> 1$, can reduce to an equation *independent of* $\mathbf{x}$ ($\frac{\partial f}{\partial t} = 0$)

$$f(x, \mathbf{V}, t) \equiv \frac{NF(\mu, \xi, t)}{T(x)^\alpha}, \quad \mu \equiv \mathbf{V} \cdot \hat{x}/\mathrm{V}, \quad \xi \equiv \left(\frac{\mathrm{V}}{\mathrm{V}_{th}} \right)^2$$

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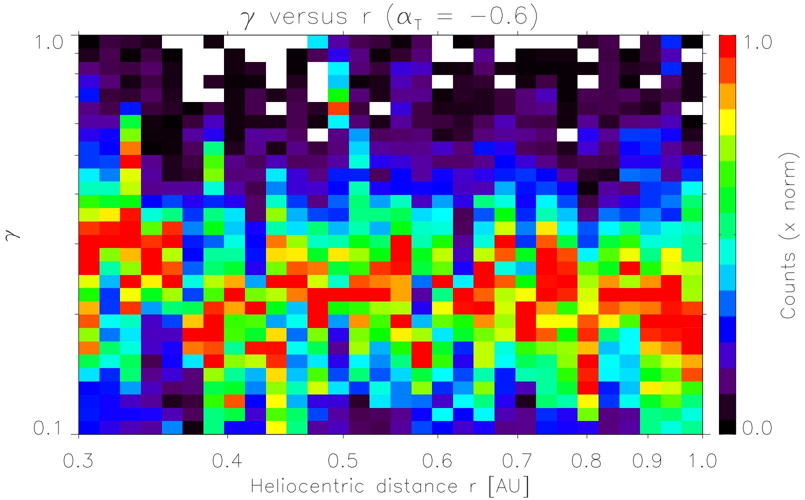
$$\gamma\Big[-\alpha\mu F - \mu\xi\frac{\partial F}{\partial \xi} + \frac{-\alpha_B}{2}(\alpha + 1/2)(1-\mu^2)\frac{\partial F}{\partial \mu}\Big] + \gamma_E\Big[\mu\frac{\partial F}{\partial \xi} + \frac{1-\mu^2}{2\xi}\frac{\partial F}{\partial \mu}\Big] + \frac{1}{\xi}\Big[\frac{\partial F}{\partial \xi} + \frac{\partial^2 F}{\partial \xi^2}\Big] + \frac{\beta}{2\xi^2}\frac{\partial}{\partial \mu}(1-\mu^2)\frac{\partial F}{\partial \mu} = 0$$

Intuitively...

- F is a normalized, dimensionless function that characterizes the "shape" of the distribution function f. $f(\mathbf{v},x)=\dfrac{n(x)F(\mathbf{v}/v_{th})}{v_{th}(x)^3}$.
- Solving for $F(\mu,\xi)$ amounts to solving for $f(\mathbf{v},x)$ throughout the system.
- $\xi \sim$energy, $\mu \sim$angle (dimensionless).
- Self-similarity is simple! Consequence of $\gamma =$constant.

Applicability (1): γ =constant?



$\gamma \propto \frac{T(dT/dr)}{n}$ plotted versus heliocentric distance $0.3 < r < 1$ AU, Helios data. (Horaites et al., 2015)

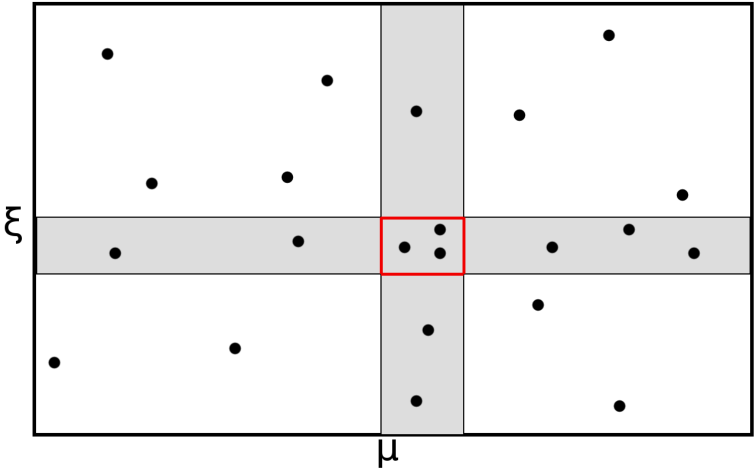
Numerical solution: Langevin Equations

Self-similar kinetic equation with our linearized collision operator is a 2nd order PDE of Fokker-Planck type. Can be converted into an equivalent set of stochastic differential equations:

$$\frac{d\xi}{d\tau} = \gamma\mu\xi^{3/2} - \gamma_E\mu\sqrt{\xi} - \frac{1}{\sqrt{\xi}} + \frac{\sqrt{2}}{\xi^{1/4}}\nu_\xi(\tau)$$

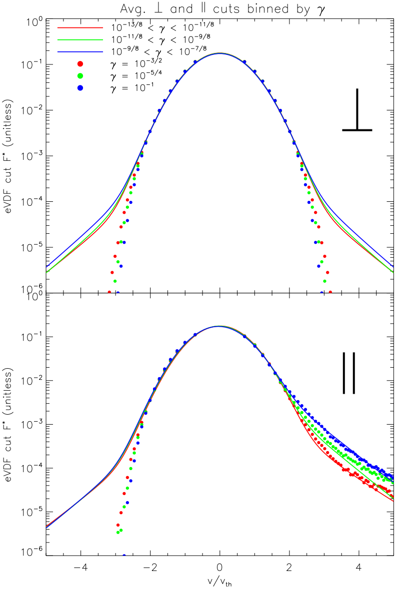
$$\frac{d\mu}{d\tau} = -(\frac{\alpha}{2} - 1)\gamma(1-\mu^2)\sqrt{\xi} - \frac{\gamma_E(1-\mu^2)}{2\sqrt{\xi}} - \frac{\beta\mu}{\xi^{3/2}} + \frac{\sqrt{\beta(1-\mu^2)}}{\xi^{3/4}}\nu_\mu(\tau)$$

Langevin Simulations



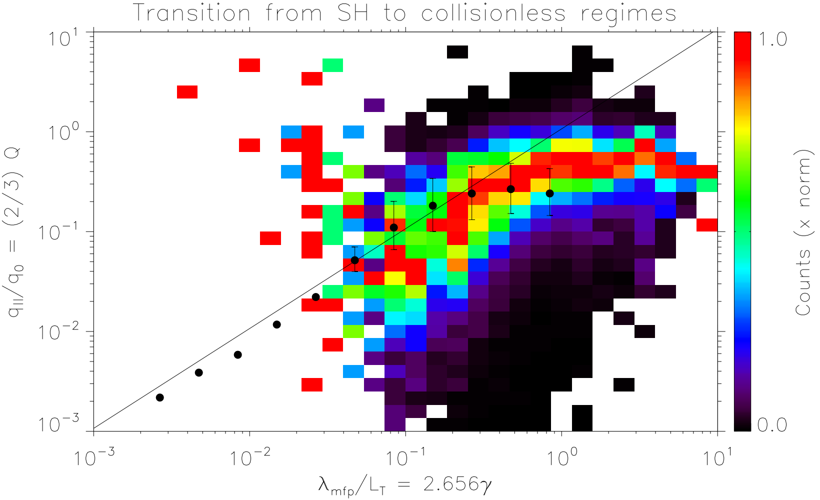
$$\frac{d\mu}{d\tau} = g(\mu, \xi, \tau),\ \frac{d\xi}{d\tau} = h(\mu, \xi, \tau)$$

eVDF Cuts



- Comparison of simulations (points) with Helios eVDF cuts (lines), ordered by γ
- High level of agreement in the core and strahl!
- Model response of the detector: Convolution

Transition from Spitzer-Härm to Collisionless limit



Latest work: Collision Operator

How can we improve the accuracy of our model? Use linearized collision operator, but don't assume $\xi >> 1$. Nonlinear Landau operator:

$$\left(\frac{\partial f_\alpha}{\partial t}\right)_{coll} \sim \frac{\partial}{\partial v_\alpha}\int d^3v'\left(f'_\beta\frac{\partial f_\alpha}{\partial v_\beta} - \frac{m_\alpha}{m_\beta}f_\alpha\frac{\partial f'_\beta}{\partial v'_\beta}\right)\left(\frac{u^2\delta_{\alpha\beta} - u_\alpha u_\beta}{u^3}\right)$$

Where $u_\alpha = v_\alpha - v'_\alpha$. Linearized operator (self-similar form):

$$\left(\frac{\partial F}{\partial \tau}\right)_{coll} = \xi^{-3/2}\left\{\frac{1}{4}\left[1 + \psi(\xi) + \psi'(\xi) - \frac{\psi(\xi)}{2\xi}\right]\frac{\partial}{\partial \mu}(1-\mu^2)\frac{\partial}{\partial \mu}F + \xi\frac{\partial}{\partial \xi}\Big[\psi(\xi)F\Big] + \xi\frac{\partial}{\partial \xi}\left[\psi(\xi)\frac{\partial F}{\partial \xi}\right]\right\}$$

Where $\psi(x) \equiv \frac{2}{\sqrt{\pi}}\int_0^x \sqrt{t}e^{-t}dt$.

Conclusions

- In the solar wind $\gamma \approx$constant, allowing self-similar kinetic equation to be applied
- Can order eVDF profiles by γ. Average Helios cuts match the results of simulations for core and strahl electron populations, but not for the halo population.
- Transition from Spitzer to collisionless regimes is predicted.