Theory and Observations of Runaway Electrons in the Solar Wind

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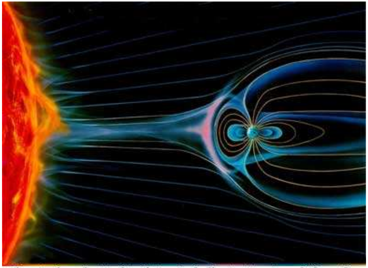
ESAC Madrid, 5/28/2019

art: Beth Racette

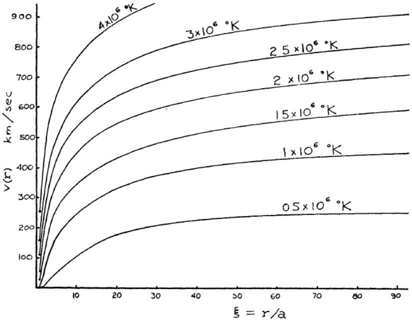
Kinetic Theory and Observations of Runaway Electrons in the Solar Wind

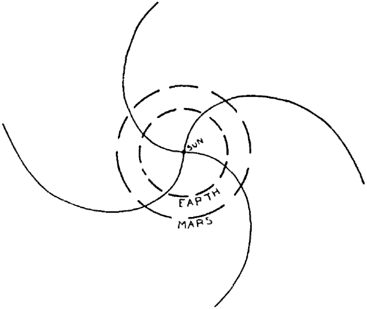
Solar Wind



Solar wind, as depicted in this artist's illustration, travels from the Sun and envelopes the Earth's magnetic field. High-energy pulses of solar wind from sunspot activity ("solar bursts" or "plasma bubbles") travel from the Sun to the Earth at speeds exceeding 500 miles per second. The pulses distort the Earth's magnetic field and produce gemagnetic storms that disrupt the Earth's environment.

The Parker Spiral (Parker, 1958)

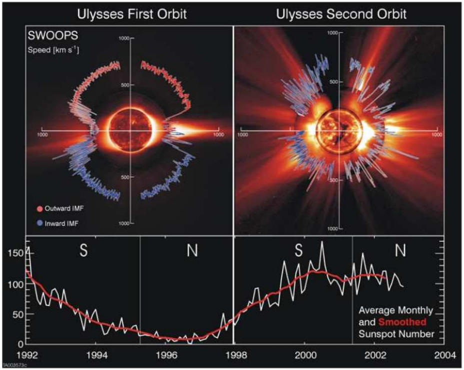




The constant outward solar wind flow, with the sun's rotation, twists the $\mathbf{B}$-field into a spiral pattern.

Fast and Slow solar wind

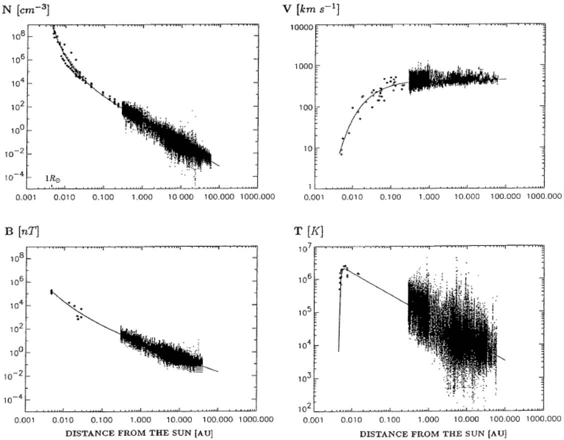
- Fast wind: $v_{sw} \gtrsim 650$ km/sec
  - low n
  - high T
  - originates from coronal holes
  - typically seen near poles
- Slow wind: $v_{sw} \sim 400$ km/sec
  - high n
  - low T
  - originates from active regions
  - typically seen in ecliptic



McComas et al., 2003

In this talk we will focus on the fast wind that is sometimes observed in the ecliptic.

Plasma Parameters vs. Distance (protons)



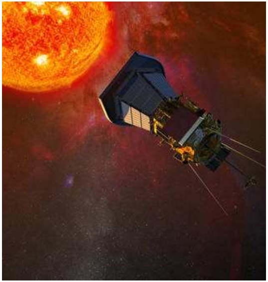
Köhnlein, 1996

- $n \sim r^{-2}$
- $v_{sw} \sim$ const.
- $T \sim r^{-2/3}$

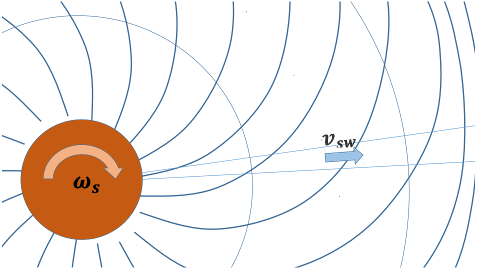
We expect heat to flow *within* the solar wind, from hot regions to cool regions.

Electron Distribution: Kinetic Physics

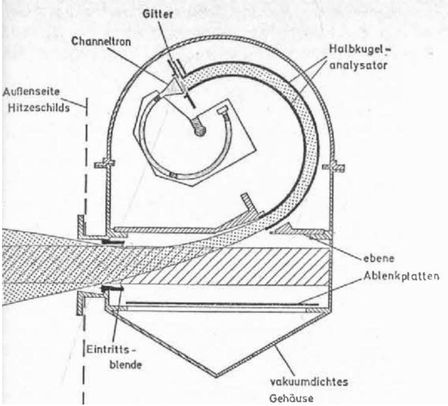
Question: What does the electron distribution function $f(\mathbf{v})$ look like in detail?



Source: JHU/APL



Measuring $f(\mathbf{v})$: Electrostatic Analyzers



Schwenn et al., 1975

https://solarsystem.nasa.gov/

- Concentric circular electrodes, radii $\approx R$
- Circular path $mv^2/R = eE \rightarrow mv^2/2 = eER/2$

Suprathermal electron populations

$$f(v_\perp, v_\parallel) = f_c + f_h + f_s$$

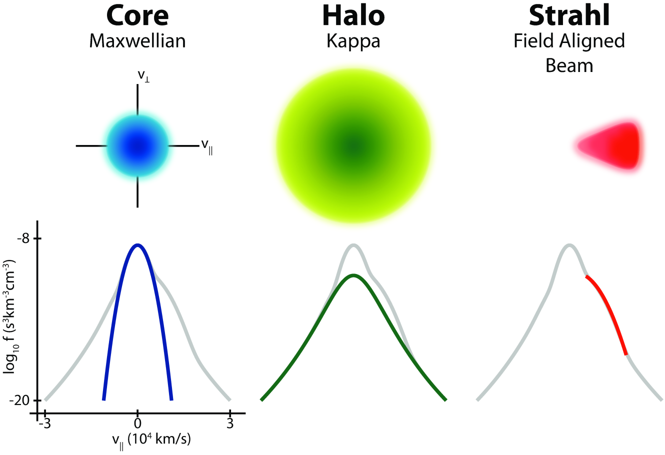


Illustration: M. Pulupa

Heat carried by electrons appears as a field-parallel *skewness* in the distribution function.

Thermal conductivity in the solar wind

Heat flux $\mathbf{q}$:

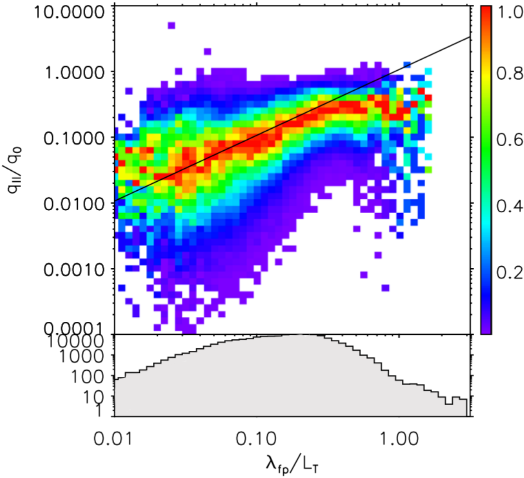
$$\mathbf{q} \equiv \frac{m_e}{2} \int \mathbf{v} v^2 f(\mathbf{v}) d^3 v$$

$$[\mathbf{q}] = \frac{W}{m^2}$$

Knudsen number γ:

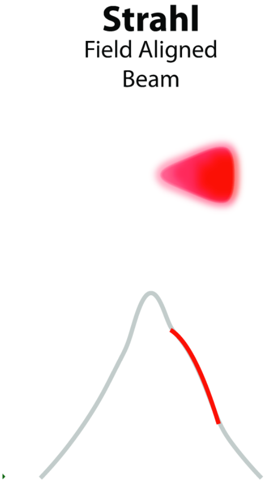
$$\gamma = \lambda_{mfp} \frac{d \ln T}{dx} = \frac{\lambda_{mfp}}{L_T}$$

- $\gamma << 1$: $\mathbf{q} = -\kappa \nabla T$ (collisional)
- $\gamma >> 1 : q \sim n T v_{th}$ (collisionless)

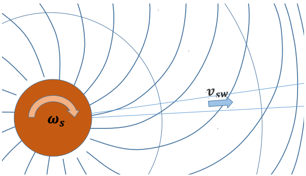


Wind data, r=1 AU (Bale et al., 2013)

Strahl: Intuitive Explanation



- Electrons focus into a beam along $\mathbf{B}$, as they try to conserve their magnetic moment ($\frac{v_\perp^2}{B}$)



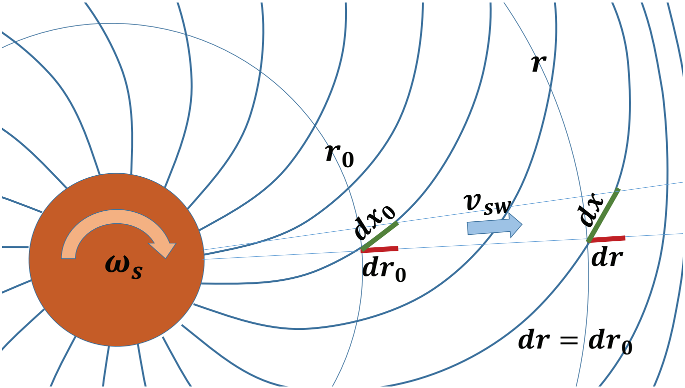
- *But,* angular diffusion (provided, e.g., by Coulomb collisions with other particles) broadens the distribution somewhat.

Kinetic Theory and Observations of Runaway Electrons in the Solar Wind

Kinetic Theory (Horaites et al., 2019)

Drift Kinetic Equation for $f(x, M, v_\parallel)$, assuming $|\mathbf{v}| >> v_{sw}$:

$$v_\parallel \hat{b} \cdot \nabla f + \left(\frac{MB(x)}{2} \nabla \cdot \hat{b} - \frac{q_e (d\phi/dx)}{m_e} \right) \frac{\partial f}{\partial v_\parallel} = \hat{C}(f)$$



Kinetic Theory (Horaites et al., 2019)

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Change variables—distance $y \propto r$ (flux freezing),
electron energy E, and magnetic moment M:

$$dy \equiv \left(\frac{16\pi e^4 \Lambda}{m_e^2 E}\right) \left(\frac{n(x)}{B(x)}\right) dx, \qquad E \equiv v^2 + (2/m_e) q_e \phi(x), \qquad M \equiv v_\perp^2 / B(x)$$

Kinetic Theory (Horaites et al., 2019)

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Strahl regime: assume large energies ($E \approx v^2$) and small angles ($\frac{MB}{E} \ll 1$). DKE then reduces to a simple form:

$$\frac{\partial}{\partial y}f(y, E, M) = \frac{\partial}{\partial M}M\frac{\partial}{\partial M}f$$

Strahl Solution

$$\frac{\partial}{\partial y} f(y, E, M) = \frac{\partial}{\partial M} M \frac{\partial}{\partial M} f$$

Solution (2D diffusion):

$$f(y, E, M) = \frac{C(E)}{y} \exp\left(-\frac{M}{y}\right)$$

Assume Parker spiral model (in the ecliptic):

$$B(r) = \frac{B(r_{45})}{\sqrt{2}} \frac{r_{45}}{r} \sqrt{1 + \frac{r_{45}^2}{r^2}},$$

where $r_{45} \approx$1 AU. Cast f in terms of r, v, and $\mu = v_{\parallel}/v$:

$$\boxed{f(r, \mu, v) = \frac{C(v^2)}{r} \exp\left\{-\frac{v^4(1-\mu^2)}{\sqrt{1 + r_{45}^2/r^2}} \left(\frac{m_e^2}{16\pi n_{45} r_{45} e^4 \Lambda}\right)\right\}}$$

Angular FWHM of Strahl

Recalling $\mu = \cos\theta$, and approximating $\sin\theta \approx \theta$, we see strahl has Gaussian angular dependence:

$$f(\theta) \propto \exp\{-A(r, r_{45})n^{-1}K^2\theta^2\},$$

where $K \equiv \frac{m_e v^2}{2}$.

The full width at half maximum, (θ_{FWHM}), is given by the formula:

$$\theta_{FWHM} \approx 24^\circ \left(\frac{K}{100\ \mathrm{eV}}\right)^{-1} \left(\frac{n(r_{45})}{5\ \mathrm{cm}^{-3}}\right)^{1/2} \left(1 + \frac{r_{45}^2}{r^2}\right)^{1/4}, \quad (1)$$

Note the scaling relations:

i For given n, $\theta_{FWHM} \propto K^{-1}$

ii For given K, $\theta_{FWHM} \propto \sqrt{n}$

Narrow strahl predicted! Need high angular resolution to detect it.

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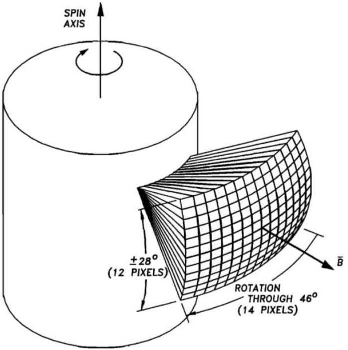
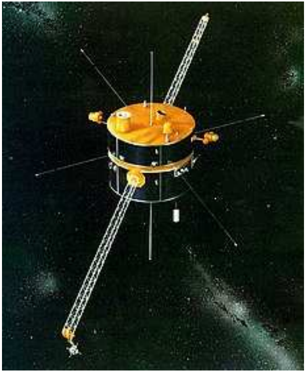
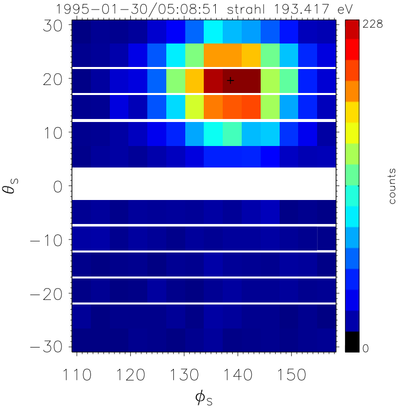
SWE Strahl Detector



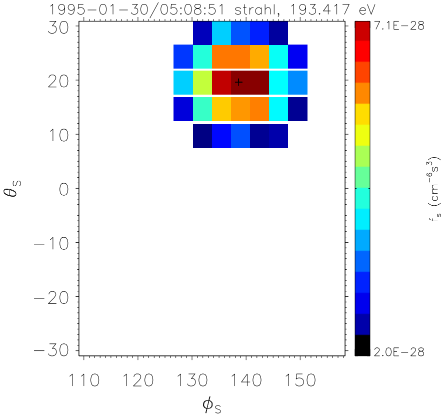


- Left: SWE strahl field of view (Ogilvie, 2000)
- Top right: SWE strahl detector (http://web.mit.edu)
- Bottom right: Wind spacecraft 1 AU (https://wind.nasa.gov)

SWE Strahl Detector



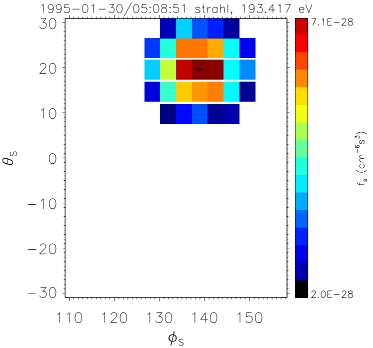
Raw counts

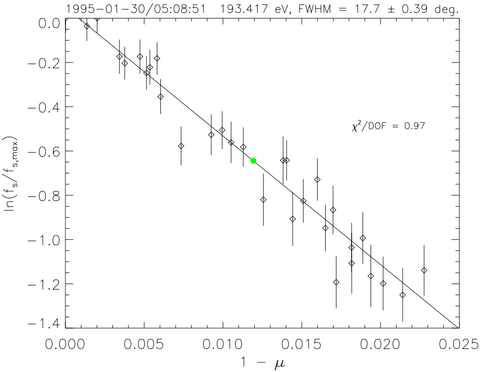


Cleaned distribution f (Horaites et al., 2018a)

Strahl electron counts measured at 3.5x4.5 degree resolution

Least squares: strahl width (Horaites et al., 2018a)

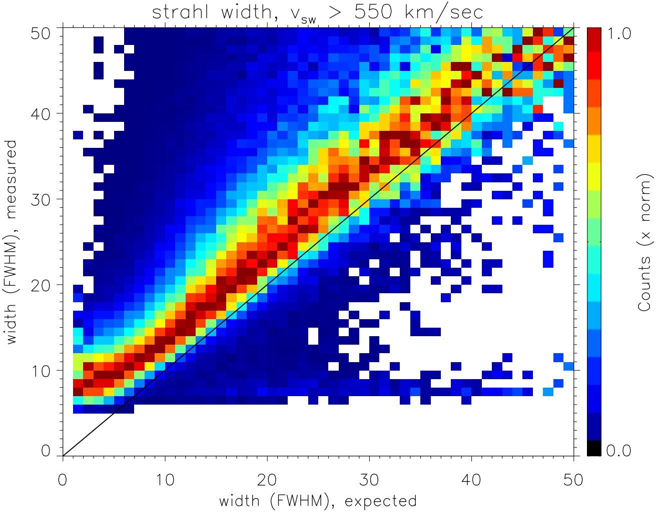




Model: $\ln\left\{\frac{f(\mu)}{f_{max}}\right\} \propto (1-\mu)$

Fit yields a "Measured θ_{FWHM}" (green dot).

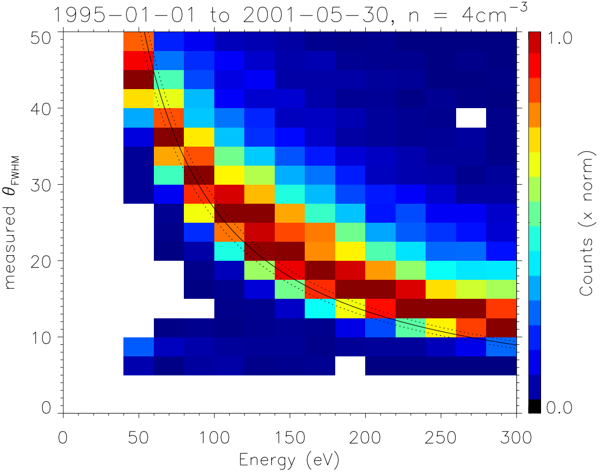
Model/Data Comparison: $v_{sw} > 550$ km/s



Horaites et al., 2019

Model/Data Comparison: $v_{sw} > 550$ km/s, $3.5 < n < 4.5$ cm^{-3}

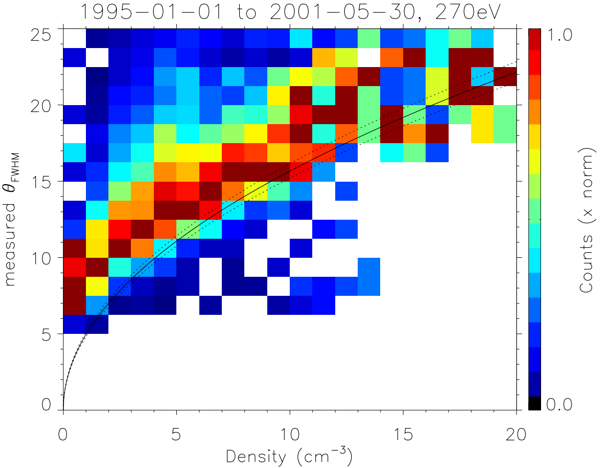
- For given n, $\theta_{FWHM} \propto K^{-1}$



Horaites et al., 2018a

Model/Data Comparison: $v_{sw} > 550$ km/s, $K = 271$ eV

ii For given K, $\theta_{FWHM} \propto \sqrt{n}$



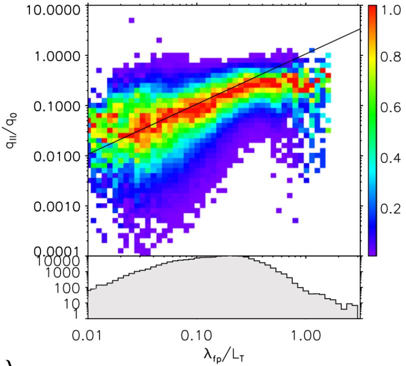
Horaites et al., 2018a

Measuring energy dependence: F_{ave}

$$f(r,\mu,v) = \frac{\boxed{C(v^2)}}{r} \exp\left\{ -\frac{v^4(1-\mu^2)}{\sqrt{1+r_{45}^2/r^2}} \left(\frac{m_e^2}{16\pi n_{45} r_{45} e^4 \Lambda} \right) \right\}$$

- Want to measure the function $C(v^2)$.
- But, $f(\mathbf{v})$ measured by SWE/strahl 1 energy at a time.
- Consider also that $\mathbf{q}$, and by extension the strahl, depends on collisionality ($\tilde{\gamma}$).

Approach: construct *averaged* distribution F_{ave} from all the strahl data, sorted by $\tilde{\gamma}$.



$$\tilde{\gamma} \equiv \frac{T^2}{2\pi e^4 \Lambda n r} \sim \frac{\lambda_{mfp}}{L_T}$$

Bale et al., 2013

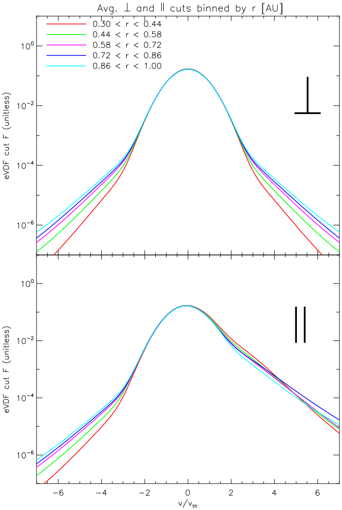
Normalized distribution function $F(\mathbf{v}/v_{th})$

- $F(\mathbf{v}/v_{th})$ is a normalized, dimensionless function that characterizes the "shape" of the distribution function $f(\mathbf{v})$.
- Introduced by Krasheninnikov (1988) in theoretical context of self-similar distributions.

$$F(\mathbf{v}/v_{th}) \equiv \frac{v_{th}^3 f(\mathbf{v})}{n}$$

Normalization:

$$\int f(\mathbf{v}) d^3 v = n, \quad \int F d^3(v/v_{th}) = 1$$



Helios F, $0.3 < r < 1$ AU (unpublished)

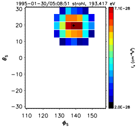
Asymptotic strahl solutions (high-energy, field-parallel)

Model distribution can be written as:

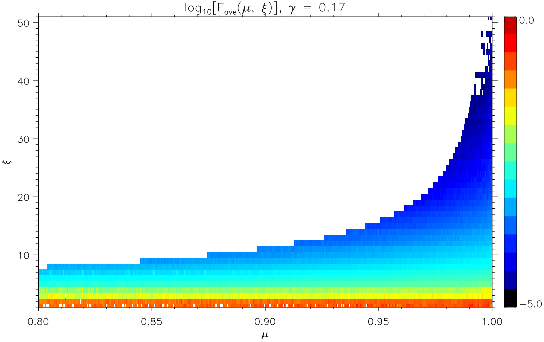
$$F(\mu, v/v_{th}) = C_0\left(\frac{v}{v_{th}}\right)^{2\epsilon} \exp\left\{\tilde{\gamma}\Omega\left(\frac{v}{v_{th}}\right)^4 (1-\mu)\right\}$$

where $\Omega \sim (-1)$

- For each spectrum, compute $\tilde{\gamma}(n)$, μ, v/v_{th}

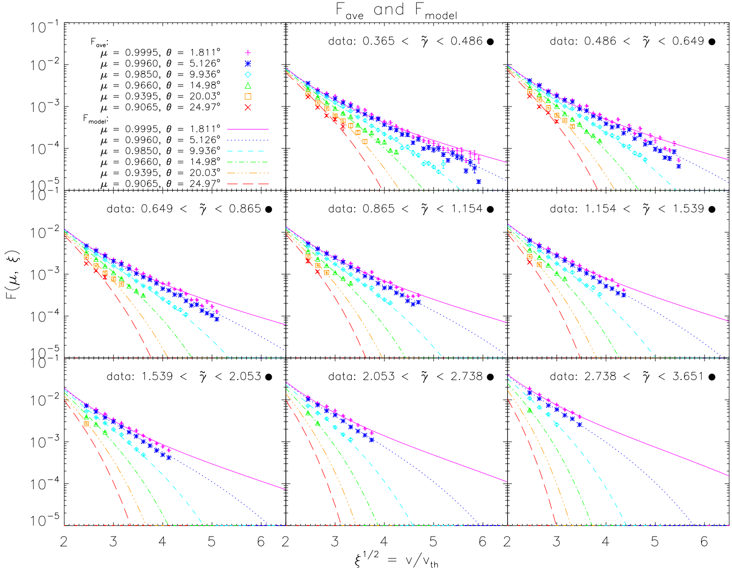


- Populate $(\mu, v/v_{th})$-space, calculate $F_{ave} \Longrightarrow$



Above: $F_{ave}(\mu, (v/v_{th})^2)$, $\tilde{\gamma} = 0.17$

F_{ave}, 2D fits $(\mu, (v/v_{th}))$



Horaites et al., 2018a

Fit Parameters

$\tilde{\gamma}$ (F_{model})	C_0	ϵ	Ω
—	model: $F(\mu, v/v_{th}) = C_0(\frac{v}{v_{th}})^{2\epsilon} \exp\{\tilde{\gamma}\Omega(\frac{v}{v_{th}})^4(1-\mu)\}$		
$\tilde{\gamma} = 0.421$	0.157 ± 0.011	-2.13 ± 0.031	-0.38 ± 0.013
$\tilde{\gamma} = 0.562$	0.191 ± 0.014	-2.14 ± 0.033	-0.35 ± 0.013
$\tilde{\gamma} = 0.749$	0.234 ± 0.019	-2.14 ± 0.035	-0.30 ± 0.012
$\tilde{\gamma} = 1.000$	0.264 ± 0.020	-2.13 ± 0.033	-0.28 ± 0.011
$\tilde{\gamma} = 1.333$	0.306 ± 0.025	-2.13 ± 0.036	-0.27 ± 0.010
$\tilde{\gamma} = 1.778$	0.401 ± 0.040	-2.19 ± 0.045	-0.25 ± 0.011
$\tilde{\gamma} = 2.371$	0.485 ± 0.053	-2.08 ± 0.050	-0.28 ± 0.010
$\tilde{\gamma} = 3.162$	0.669 ± 0.065	-2.02 ± 0.046	-0.28 ± 0.009

Note: C_0 gives strahl amplitude, relative to the core.

Q: is this distribution stable?

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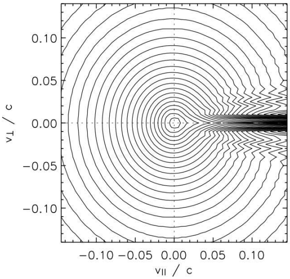
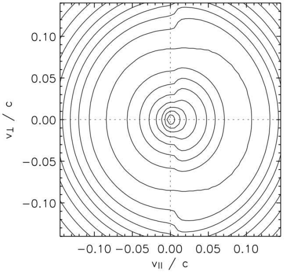
Anomalous Scattering of the Strahl

Some models propose that the strahl is scattered by wave-particle interactions.
Candidate waves:

- Whistler (e.g., Vocks et al., 2005, pictured)
- Langmuir (e.g., Seough et al., 2015)

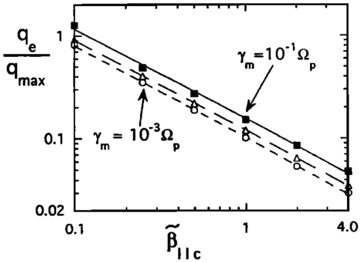


without whistlers

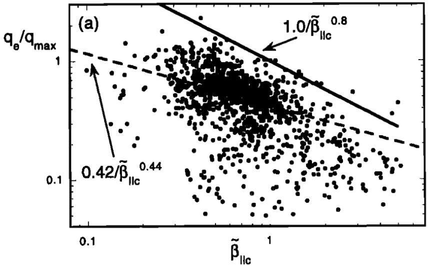


with whistlers

Whistler Heat Flux Instability



Gary et al., 1994

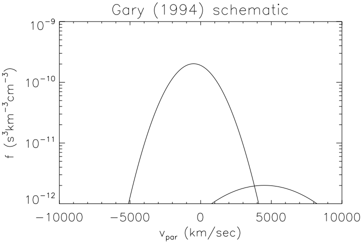


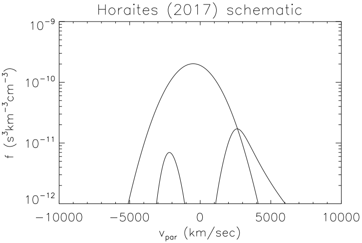
Gary et al., 1999

$$q_{max} \sim nTv_{th}$$

Whistler Heat Flux Instability

Gary et al., (1994) proposed a model, where the electrons are described by 2 drifting Maxwellians.





Core drift v_c follows from current balance:

$$\sum_\sigma J_\sigma = 0 \rightarrow v_c = -J_s/n_c$$

How will stability analysis change if we model the strahl more realistically?

Core-strahl model (Horaites et al., 2018b)

Model distribution function as sum of core and strahl components:

$$f = f_c + f_s$$

Core distribution:

$$f_c(\mu, v) = \frac{n_c}{\pi^{3/2} v_{th}^3} \exp\left(\frac{-v^2 + 2\mu v v_c - v_c^2}{v_{th}^2}\right).$$

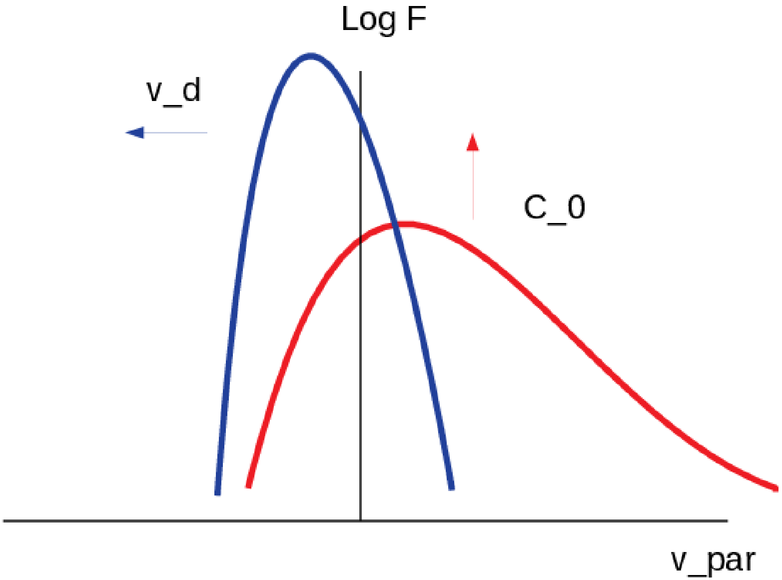
Strahl distribution:

$$f_s(\mu, v) = \boxed{A(v)} \frac{n_c}{v_{th}^3} C_0 \left(\frac{v}{v_{th}}\right)^{2\epsilon} \exp[\tilde{\gamma}\Omega(v/v_{th})^4(1-\mu)],$$

where we define a truncation function $A(v)$, with $a = 10$, $b = 2\epsilon - 4$:

$$A(v) = \left(\frac{1}{1 + a(v/v_{th})^b}\right).$$

Require $J_{\parallel} = \int f v_{\parallel} d^3 v = 0$



Dispersion Relation Solver

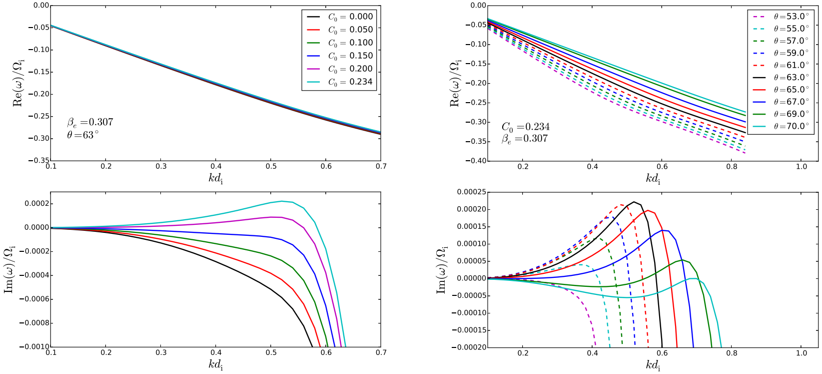
We use the kinetic dispersion relation solver, LEOPARD (Astfalk et al., 2017).

- solves kinetic equation for linear waves in magnetized plasma
- allows for arbitrary (gyrotropic) distribution functions
- can solve for modes with arbitrary propagation angle
- requires an initial guess for $\omega(\mathbf{k}) \rightarrow$ search magnetosonic, kinetic alfven, and whistler branches

Code computes $\epsilon_{ij}(\omega, \mathbf{k})$ and solves for dispersion relation $\omega(\mathbf{k})$ from:

$$\left\{ k^2 \delta_{ij} - k_i k_j - \frac{w^2}{c^2} \epsilon_{ij}(\omega, \mathbf{k}) \right\} E_j = 0.$$

KAW Instability

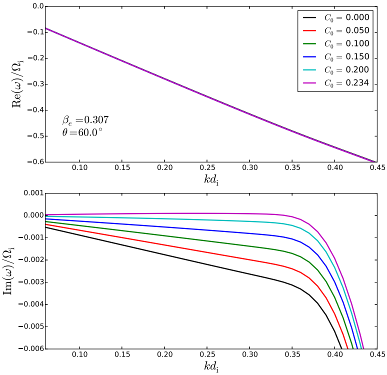


Im(ω)↑ as C_0 ↑.

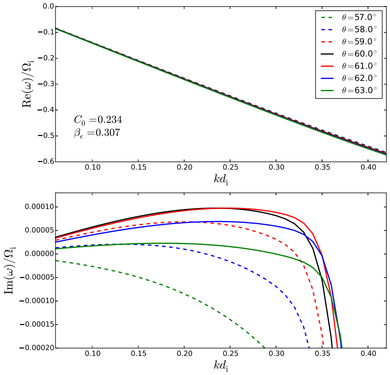
Max. growth rate at $\theta \approx 63^\circ$.

$$\Omega_i = \frac{eB}{m_p}, \qquad d_i = \frac{v_A}{\Omega_i}$$

Magnetosonic Instability



$\mathrm{Im}(\omega)\uparrow$ as $C_0 \uparrow$.



Max. growth rate at $\theta \approx 60^\circ$.

$$\Omega_i = \frac{eB}{m_p}, \qquad d_i = \frac{v_A}{\Omega_i}$$

Resonance

Instabilities found, but which particles are affected?
Resonance condition:

$$\omega - k_\parallel v_\parallel = n\Omega_e$$

Assume $v_A \approx v_{th,i}$, and $T_i \approx T_e$ (typical conditions at 1 AU). From dispersion solver, $0.4 < kd_i < 0.6$, $0.3 \lesssim |\omega/\Omega_i| < 0.6$ Rearrange, neglecting small terms:
If $|n| > 0$,

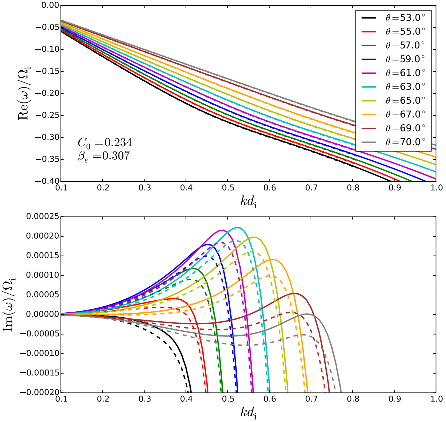
$$|v| \approx \frac{|n|\sqrt{m_i/m_e}}{k_\parallel d_i} v_{th,e} \gtrsim 50 v_{th,e}$$

If $n = 0$,

$$|v| = v_A \frac{|\omega/\Omega_i|}{k_\parallel d_i} < v_A \ll v_{th,e}$$

Resonances do not fall in strahl regime! ($2v_{th,e} \lesssim v \lesssim 10 v_{th,e}$)!
Also note for $n = 0$, resonant velocities are negative ($\omega/\Omega_i < 0$).

Isotropic halo damps growth (KAW)



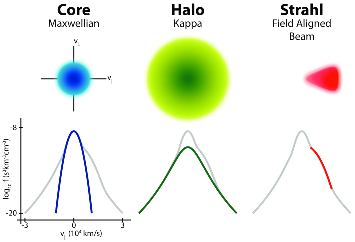
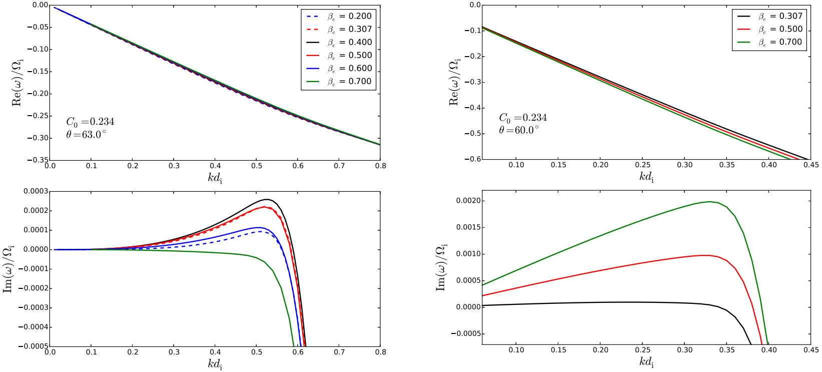


Illustration: M. Pulupa

with halo (dashed), without (solid)

Variation with β_e



KAW: as $\beta_e \uparrow$, $Im(\omega)$ reaches a maximum then stabilizes.

MS: as $\beta_e \uparrow$, $Im(\omega) \uparrow$.

β_e is smaller near the sun than at 1 AU, so MS instability may be less important at small heliocentric distances.

Kinetic Theory and Observations of Runaway Electrons in the Solar Wind

Conclusions

- We develop a new model of the strahl (runaway) electron population in the solar wind, which incorporates the magnetic focusing and Coulomb scattering of the distribution.
- Our model accurately describes the data at 1 AU, as measured by the Wind satellite.
- Linear analysis shows two growing modes at 1 AU: kinetic alfven and magnetosonic. Resonate with sunward-drifting core electrons, with phase velocities $\omega/k_{\|} \sim v_A$.
- No whistler instability was found.

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