Self-Similar Kinetic Theory and Application to the Solar Wind

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Thermal conductivity in weakly collisional plasma

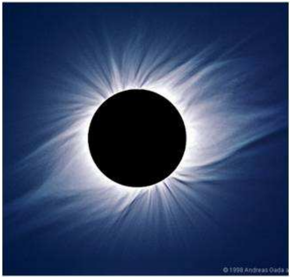
Knudsen number:

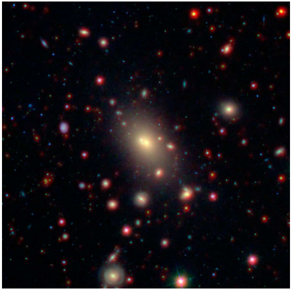
$$\gamma = \lambda_{mfp} \frac{d \ln T}{dx} \propto \frac{T dT/dx}{n}$$

heat flux:

$$\mathbf{q} \equiv \frac{m_e}{2} \int \mathbf{v} v^2 f(\mathbf{v}) d^3 v$$

- $\gamma << 1$: $\mathbf{q} = -\kappa \nabla T$ (collisional)
- $\gamma >> 1 : q \sim nTv_{th}$ (collisionless)
- $0.01 \lesssim \gamma \lesssim 1 : \mathbf{q} = ???$ (weakly collisional)





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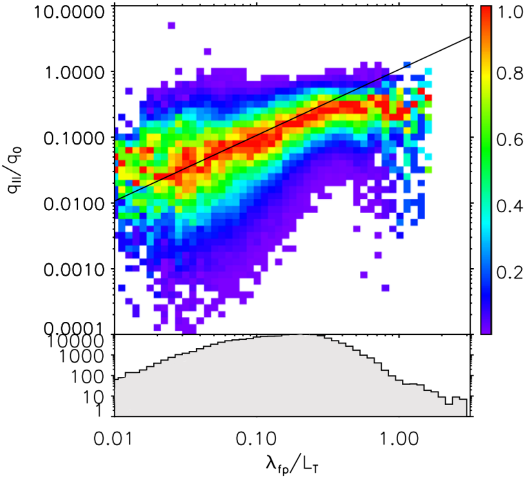

Wind data, r=1 AU (Bale, 2013)

Self-similar Kinetic Theory

Drift Kinetic Equation ($|\vec{V}| >> \mathrm{V}_{sw}$):

$$\frac{\partial f}{\partial t} + \mathrm{V}_{\|}\hat{b} \cdot \nabla f + \left(\mu_B B \nabla \cdot \hat{b} + \frac{q_e E_{\|}}{m}\right)\frac{\partial f}{\partial \mathrm{V}_{\|}} = \hat{C}(f)$$

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If Knudsen number $\gamma \sim \frac{\lambda_{mfp}}{L_T}$ =constant, then for $\frac{\mathrm{V}}{\mathrm{V}_{th}} >> 1$, can reduce to an equation *independent of* $\mathbf{x}$ ($\frac{\partial f}{\partial t} = 0$)

$$f(\mathbf{x},\mathbf{V},t) \equiv \frac{NF(\mu,\xi,t)}{T(\mathbf{x})^{\alpha}}, \qquad \mu \equiv \mathbf{V}\cdot\hat{x}/\mathrm{V}, \qquad \xi \equiv \left(\frac{\mathrm{V}}{\mathrm{V}_{th}}\right)^2$$

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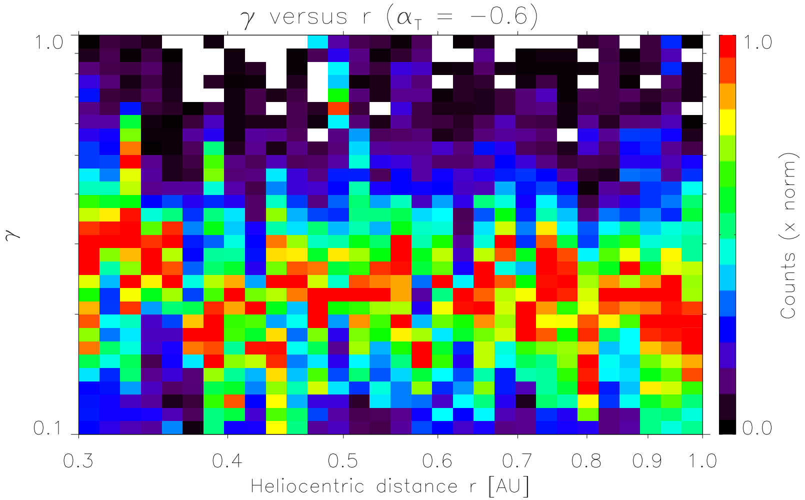
$$\gamma\Big[-\alpha\mu F - \mu\xi\frac{\partial F}{\partial \xi} + \frac{-\alpha_B}{2}(\alpha+1/2)(1-\mu^2)\frac{\partial F}{\partial \mu}\Big] + \gamma_E\Big[\mu\frac{\partial F}{\partial \xi} + \frac{1-\mu^2}{2\xi}\frac{\partial F}{\partial \mu}\Big] + \frac{1}{\xi}\Big[\frac{\partial F}{\partial \xi} + \frac{\partial^2 F}{\partial \xi^2}\Big] + \frac{\beta}{2\xi^2}\frac{\partial}{\partial \mu}(1-\mu^2)\frac{\partial F}{\partial \mu} = 0$$

Applicability

Three things to check before applying self-similar theory:

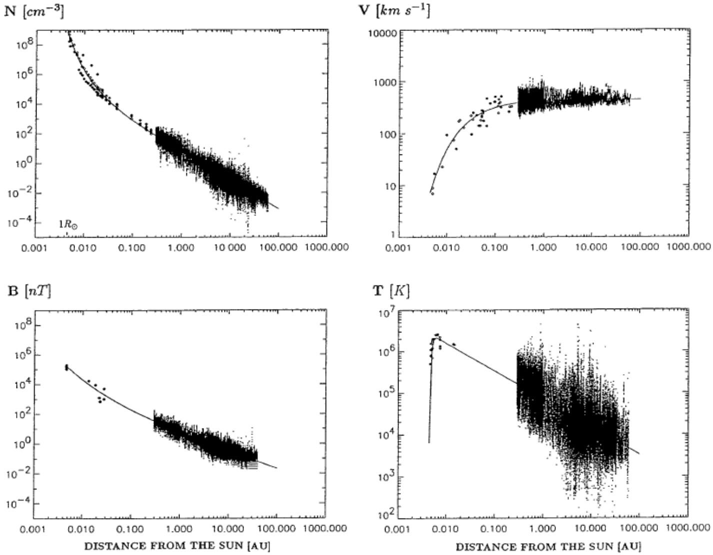
- $\gamma(x) =$ constant?
- n, T, B, q, vary as power laws?
- Do electron distributions actually exhibit self-similarity?

Applicability (1): γ =constant?



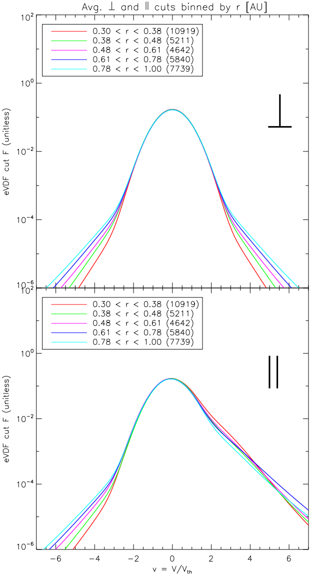
$\gamma \propto \frac{T(dT/dr)}{n}$ plotted versus heliocentric distance $0.3 < r < 1$ AU. (Helios electron data)

Applicability (2): Power Laws $X \propto r^{\alpha_X}$

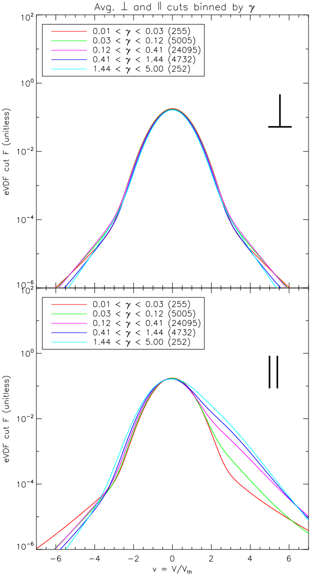


Köhnlein, 1996

Applicability (3): Self-similarity $F(\vec{v}/v_{th}) = f(\vec{v})\frac{v_{th}^3}{n}$



$0.3 < r < 1$ AU



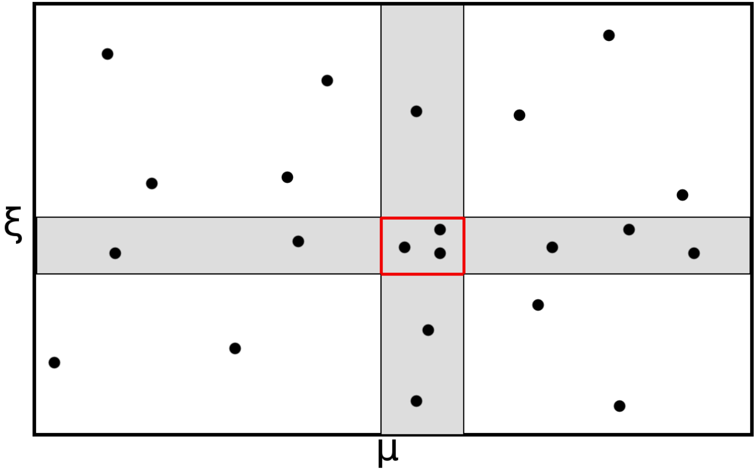
$0.01 < \gamma < 5$

Numerical Solution: Langevin Equations

Self-similar kinetic equation with our linearized collision operator is a 2nd order PDE of Fokker-Planck type. Can be converted into an equivalent set of stochastic differential equations:

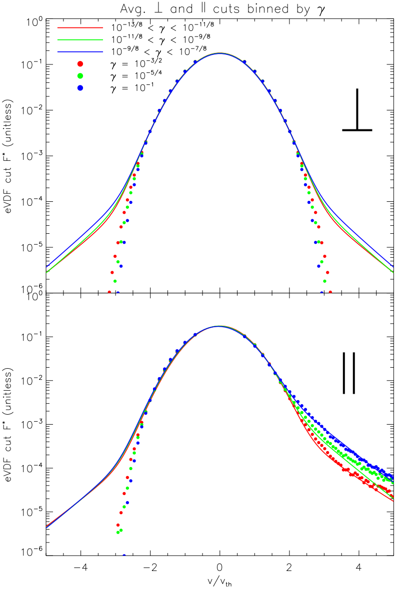
$$
\begin{aligned}
\frac{d\xi}{d\tau} &= \gamma\mu\xi^{3/2} - \gamma_E\mu\sqrt{\xi} - \frac{1}{\sqrt{\xi}} + \frac{\sqrt{2}}{\xi^{1/4}}\nu_\xi(\tau) \\
\frac{d\mu}{d\tau} &= \frac{5}{2}\gamma(1-\mu^2)\sqrt{\xi} \\
&- \frac{\gamma_E(1-\mu^2)}{2\sqrt{\xi}} - \frac{\beta\mu}{\xi^{3/2}} + \frac{\sqrt{\beta(1-\mu^2)}}{\xi^{3/4}}\nu_\mu(\tau)
\end{aligned}
$$

Langevin Simulations



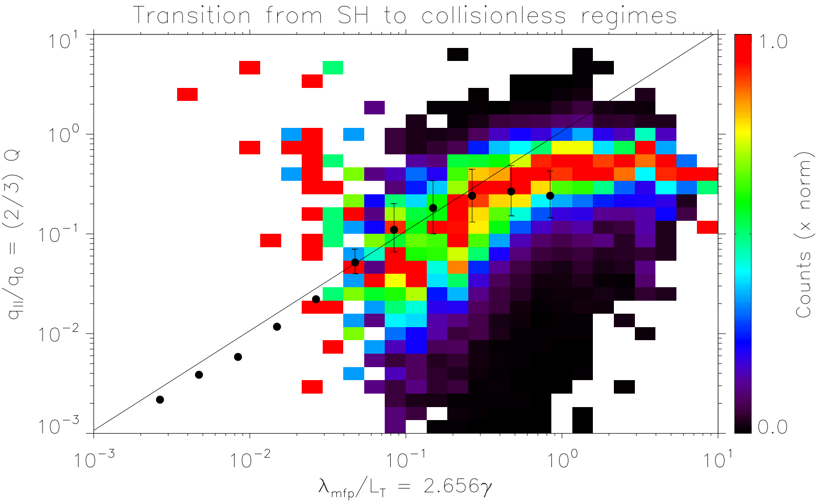
$$\frac{d\mu}{dt} = g(\mu, \xi, t),\ \frac{d\xi}{dt} = h(\mu, \xi, t)$$

eVDF Cuts



- Comparison of simulations (points) with Helios eVDF cuts (lines), ordered by γ
- High level of agreement in the core and strahl!
- Model response of the detector: Convolution

Transition from Spitzer-Härm to Collisionless limit



Conclusions

- In the solar wind $\gamma \approx$constant, allowing self-similar kinetic equation to be applied
- Can order eVDF profiles by γ. Average Helios cuts match the results of simulations for core and strahl electron populations, but not for the halo population.
- Transition from Spitzer to collisionless regimes is predicted.