# Application of Self-Similar Kinetic Theory to the Solar Wind: Data and Simulations

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# Theory: Background

Drift Kinetic Equation (ignore $\mathbf{E} \times \mathbf{B}$ drifts):

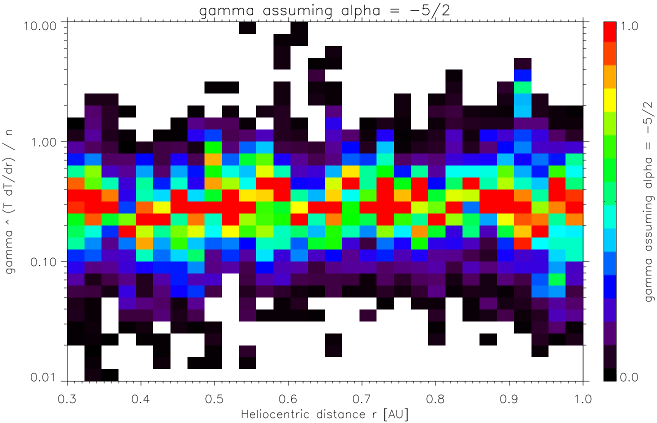
$$\frac{\partial f}{\partial t} + \mathsf{V}_{\parallel} \hat{b} \cdot \nabla f + \Big( \mu_B B \nabla \cdot \hat{b} + \frac{q_e E_{\parallel}}{m} \Big) \frac{\partial f}{\partial \mathsf{V}_{\parallel}} = C(f) \qquad (1)$$

If Knudsen number (usually denoted Kn) $\gamma = \frac{\lambda_{mfp}}{L_T}$ =constant, then for $\mathsf{v} \equiv \frac{\mathsf{V}}{\mathsf{V}_{th}} >> 1$, reduces to an equation *independent of* $\mathbf{x}$

$$f(\mathbf{x}, \mathbf{V}, t) \equiv \frac{N F(\mu, \xi, t)}{T(\mathbf{x})^{\alpha}}, \mu \equiv \cos\theta, \xi \equiv \Big( \frac{\mathsf{V}}{\mathsf{V}_{th}} \Big)^2 \qquad (2)$$

$$\begin{aligned} \frac{\partial F(\mu, \xi, t)}{\partial t} = \nu \xi^{1/2} \Big\{ & \gamma \Big[ -\alpha \mu F - \mu \xi \frac{\partial F}{\partial \xi} + \frac{-\alpha_B}{2} (\alpha + 1/2)(1 - \mu^2) \frac{\partial F}{\partial \mu} \Big] + \\ & \gamma_E \Big[ \mu \frac{\partial F}{\partial \xi} + \frac{1 - \mu^2}{2\xi} \frac{\partial F}{\partial \mu} \Big] + \\ & \frac{1}{\xi} \Big[ \frac{\partial F}{\partial \xi} + \frac{\partial^2 F}{\partial \xi^2} \Big] + \frac{\beta}{2\xi^2} \frac{\partial}{\partial \mu} (1 - \mu^2) \frac{\partial F}{\partial \mu} \Big\} \end{aligned} \qquad (3)$$

# Applicability: $\gamma = \frac{\lambda_{mfp}}{L_T}$ =constant



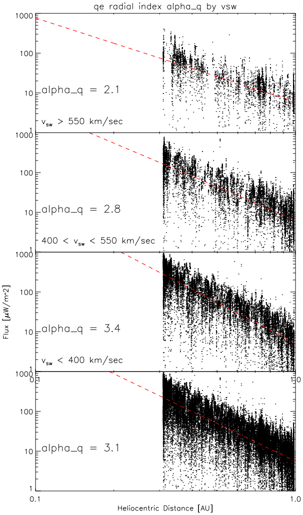
$\gamma \propto \frac{T(dT/dr)}{n}$ plotted versus heliocentric distance $0.3 < r < 1$ AU. (Helios electron data)

# Applicability: Power Laws $X \propto r^{\alpha_X}$

n, T, q, B go as power laws in solar wind. Choose $\alpha_n$ and $\alpha_T$, $\alpha_q$ are specified.

| - | $\alpha_{expected}$ | $\alpha_{observed}$ |
|---|---|---|
| n | -2 | -2.24 |
| T | -0.5 | -0.56 |
| q | -2.75 | -3.06 |
| B | any | -1.6 |

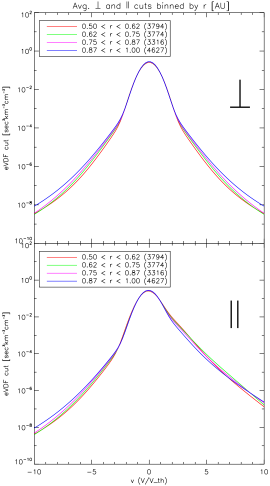
Theory matches well! Observational values taken from fits to Helios data $0.3 < r < 1$ AU.



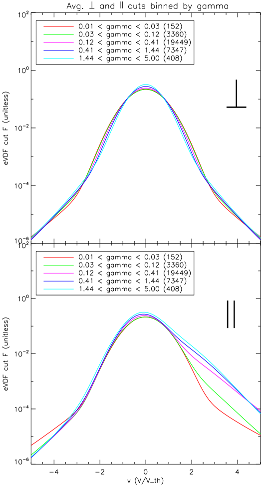
Example of power laws: electron heat flux $q_{\|}(r)$

# Applicability: Helios fits

Can normalize Helios fits by formula $F \equiv \frac{f(\mathbf{x},\mathbf{V})T(\mathbf{x})^{\alpha}}{N}$ to get the self-similar distribution function $F$



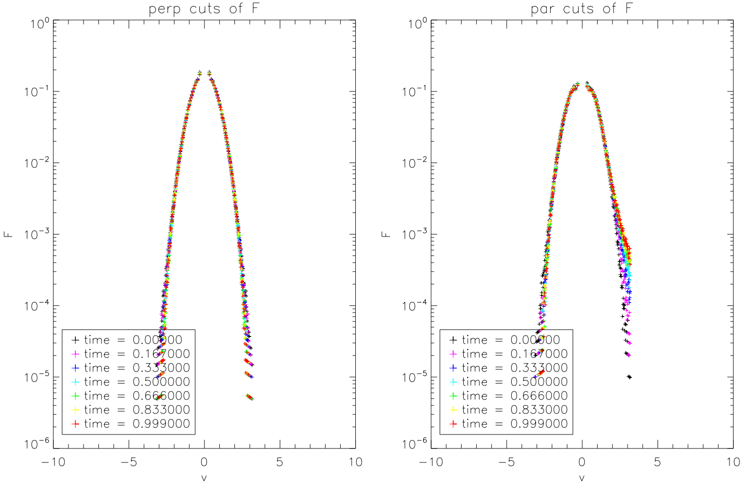
$0.5 < r < 1$ AU



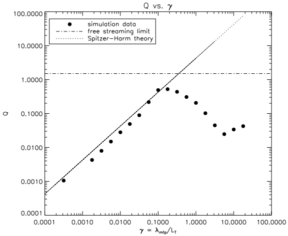
$0.01 < \gamma < 5$

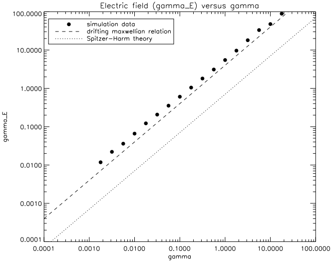
# Langevin Simulations

Simulate time-dependent kinetic equation, by deriving stochastic *Langevin equations*. Populate phase space $(\mu, \xi)$ with $N_p$ particles, and as $N_p \to \infty$, exact solution is obtained. Below: cuts versus time, $\gamma = 0.05$, $N_p = 1e7$



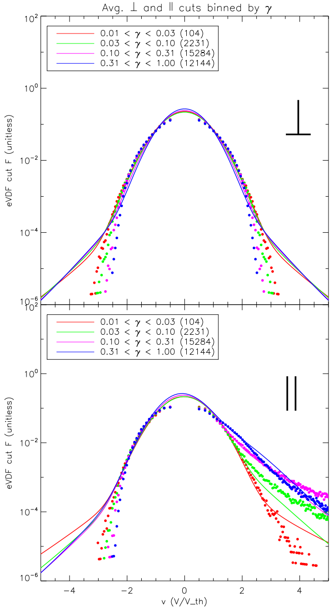
# Comparison with Spitzer theory





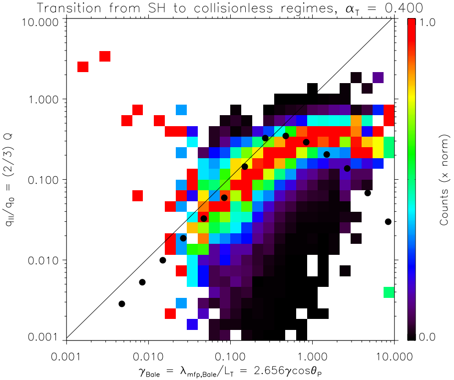
- $Q \equiv \int F \mathrm{v}_{\|} \mathrm{v}^2 d^3\mathrm{v}$
- Follows Spitzer-Härm relation $Q_{SH} \propto \gamma$ for $\gamma << 1$
- Transitions to collisionless heat flux at $\gamma \approx 0.1$
- Magnitude of Q depends on choice of $\mathrm{v}_{max}$
- Electric field follows Spitzer-Härm scaling $\frac{E}{E_D} \propto \gamma$ in both regimes
- Can simulations be made to match theory exactly?

# eVDF Cuts



- Comparison of simulations with Helios eVDF cuts averaged into bins ordered by $\gamma$
- $\gamma$ are logarithmically spaced $0.01 < \gamma < 1$
- High level of agreement in the core and strahl!
- Less agreement in the halo... not enough points in simulation?
- Sharp peaks in Langevin simulation and Helios data are smeared out, due to sampling in phase space

# Transition from Spitzer-Härm to Collisionless limit



- Histogram of $\frac{q_\parallel}{q_0}$, where $q_0 \equiv \frac{3}{2} n \mathrm{V}_{th} T$, vs. $\gamma$ (see Bale, 2013)
- Langevin simulations (dots) match the data well
- Departure from expected form for $\gamma > 1$, probably because our collision operator doesn't apply for strongly non-Maxwellian core

# Conclusions

- In the solar wind $\gamma \approx$constant, allowing self-similar kinetic equation to be applied
- Can order eVDF profiles by $\gamma$. Average Helios cuts match the results of simulations in core and strahl electron populations, but agreement with halo is as yet undetermined.
- Transition from Spitzer to collisionless regimes is correctly predicted, although there may be some issues with limits on validity of the theory