Quantum Mechanics, Physics 531
Homework Assignment 5, due April 13, 2009

Problem 1. Find the commutator $\hat{K}_0 = [\hat{K}_+; \hat{K}_-]/2$ of the operators $\hat{K}_\pm = \hat{a}_\pm \hat{a}_\pm / 2$, where the operators $\hat{a}_\pm$ are the raising and lowering operators of a harmonic oscillator, their commutator is $[\hat{a}_-; \hat{a}_+] = 1$.

Calculate the commutation relations $[\hat{K}_\pm; \hat{K}_0]$.

Problem 2. Problem 4.31.

Problem 3. Problem 4.49.

Problem 4. Problem 4.38.

Problem 5. Calculate energies E_n of stationary states of a two-dimensional particle moving in a potential

$$U(\boldsymbol{r}) = \frac{\hbar^2}{2m}\left[\alpha^2 |\boldsymbol{r}|^2 + \frac{\beta^2}{|\boldsymbol{r}|^2}\right].$$

Use the cylindrical coordinates and look for the wave function in the form

$$\psi_n(|\boldsymbol{r}|, \varphi) = e^{im_z \varphi} e^{-\alpha |\boldsymbol{r}|^2/2} |\boldsymbol{r}|^{\sqrt{\beta^2 + m_z^2}} w(|\boldsymbol{r}|),$$

where $w(|\boldsymbol{r}|)$ is a finite order polynomial, $w(0) \neq 0$. Explain the choice of the above equation for the wave function $\psi_n(|\boldsymbol{r}|, \varphi)$.

Problem 6. Problem 7.2.

Problem 7. Problem 7.7.