Solid State Physics, Physics 551
Final Exam, May 12, 2010 at 2:25 pm

Problem 1. A monovalent metal consists of positive ion cores placed in a uniform cloud of electrons. The average energy per atom is

$$E(a) = -\alpha\frac{e^2}{a} + \beta\frac{\hbar^2}{ma^2},$$

where a is the size of a cube containing a single electron and α, β are dimensionless parameters. At $T = 0$, calculate a) equilibrium value of a, b) the bulk compression modulus $B = -V(\partial P/\partial V)$, and c) the velocity of sound c. *Hint:* Pressure P can be calculated as a derivative of total energy with respect to volume, or $P = -\partial E/\partial V_e$ with $V_e = a^3$.

Problem 2. a) For a metal, schematically draw the temperature dependence of the specific heat contributions of conduction electrons C_e and phonons C_p for temperatures T ranging from absolute zero to metal melting temperature.

b) Place the Fermi energy E_F and the Debye frequency ω_D on the temperature axis T. Indicate the characteristic temperature regimes for the two contributions.

c) Estimate the ratio C_e/C_p for the temperature of liquid Helium and at the room temperature.

d) Show dependence of the Fermi energy and the Debye frequency on number density of atoms.

Problem 3. Consider a system of two types of charge carriers in the Drude model. The two carriers have the same density n, opposite charges $\pm e$ and their masses and relaxation rates are $m_\pm$ and $\tau_\pm$. Find the conductivity matrix for this system. Calculate the magnetoresistance, $\Delta\rho = \rho_{xx}(B) - \rho_{xx}(B = 0)$ and the Hall coefficient R_H. Specify which parameters define the sign of $\Delta\rho$ and R_H.

Problem 4. A very thick plate of type-I superconductor has the critical field $H_c = 1$T. It is found that a film of $\delta = 0.5\mu$m thickness has a critical field of $H_c(0.5\mu\text{m}) = 1.1$T. calculate the magnetic penetration depth λ. Find the thickness δ_2 of the film with critical field $H_c(\delta_2) = 2$T. Assume that the penetration depth is given by the London theory and is independent of the magnetic field. Also, disregard the demagnetization effects.

Problem 5. The free energy of a ferromagnet is a function of its magnetic moment M:

$$F = A(T - T_c)M^2 + BM^4 - HM,$$

where T is its temperature, H is the strength of an applied magnetic field, coefficients A and B and the transition temperature T_c can be considered as fixed parameters.

a) Sketch dependence of magnetic moment on magnetic field for $T < T_c$ and $T > T_c$;

b) Find the asymptotic behavior of the magnetic moment M in a strong magnetic field H;

c) Calculate the width $H_r - H_l$ of the hysteresis loop and evaluate dependence of its area on $T_c - T$. What is the physical meaning of the area of the hysteresis loop?