Statistical Mechanics, Physics 715
Homework Assignment 3, due November 23, 2009

Problem 1. Problem 12.1 from Huang, 2nd edition.

Problem 2. Thermodynamic properties of an isotropic dielectric crystal at temperatures above the Debye temperature are dominated by phonon contribution. The free energy of phonons in the high-temperature limit is given by

$$F = N\epsilon_0 - 3NT \ln\left(T/\hbar\bar{\omega}\right). \tag{1}$$

a) Calculate energy E of the crystal, its entropy S and heat capacity C.

b) Determine the maximal work, which can be obtained from two identical solid bodies at initial temperatures T_1 and $T_2 > T_1$, in the process when their temperatures become equal. Hint: use expressions for the entropy S and internal energy E from part (a) of this problem.

Problem 3. Find the first two non-vanishing terms in a series in powers of temperature for the specific heat of an ideal fermi gas in a three dimensional space.

Problem 4. Consider a relativistic degenerate fermion gas with energy $\varepsilon(p) = cp$ in $d-$dimensions ($d = 2, 3$).
a) Find the Fermi energy $E_{\rm F}$ of the gas as a function of the particle concentration N/V.
b) Find the relation between the total energy of the gas and its pressure P at arbitrary temperature.
c) Calculate the lowest order correction to the chemical potential and total energy E at finite temperature $T \ll E_{\rm F}$.
d) What is the specific heat C_V?

Problem 5. Consider an ideal Bose gas of non-relativistic particles ($E(p) = p^2/2m$) in four dimensional space.
a) Demonstrate that the Bose-Einstein condensation occurs and find the corresponding temperature of the transition $T_{\rm BEC}$ as a function of the number of particles N and 4-dimensional volume W.
b) Find the specific heat at fixed N and W as a function of $T < T_{\rm BEC}$.
c) Find the chemical potential μ just above the transition temperature $T > T_{\rm BEC}$.

Problem 6. Consider a large number N of atoms of mass m trapped in a spherical harmonic trap with potential $U(r) = Kr^2/2$. Find the dependence of the Bose-Einstein condensation temperature $T_{\rm BEC}$ on N and the dependence of the condensate fraction N_0 on $T_{\rm BEC}$ and temperature T.

Problem 7. Following Section 8.5 of Huang, derive expression for the entropy of a gas of boson or fermions, Eq. (8.47).

Problem 8. Calculate magnetic moment of a two-dimensional degenerate electron gas in a perpendicular magnetic field B at finite temperature T. Assume that electrons have the effective mass m^* and the g factor different from those of free electrons.