Spin relaxation in quantum dots due to e^- exchange with leads

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ABSTRACT

- Quantum dot is a basis for many nano-technological applications. Electronic and magnetic (spin) properties of *few-electron* systems are of a particular interest;
- Traditionally considered e^- spin relaxation channels in a quantum dot are *intrinsic*: associated with a) hyperfine coupling to nuclei and b) spin-orbit coupling in the dot;
- We explore an alternative spin-flip process that is due to electron exchange between dot and an electrode. We present a quantitative description of this spin-flip mechanism and the resulting current in a double dot system. We find values for the relaxation times that may be much shorter than those due to intrinsic mechanisms, and thus dominate the relaxation;

1 Introduction

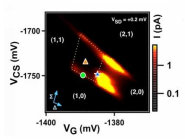
1.1 Spin relaxation in quantum dots

- GaAs
  - HF – Hyperfine coupling to nuclear spins
  - SO – Spin-orbit interaction (Rashba etc.)
- Intrinsic spin-flip times T_1 (Hanson *et al.* [1], Zutić *et al.* [2])

  $10-100\ ns$ $(B=0)$
  $100\ \mu s - 200\ ms$ $(B\neq 0)$

  can be quite long!

- Si
  - HF and SO mechanisms are suppressed - relaxation times up to 1 second!
  - Relevant experiment: double dots

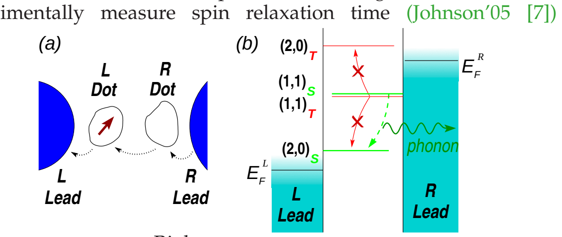


N.Shaji *et al.* . [3]

- Blockade region – measure of spin relaxation
- Peaks – *resonant* spin exchange with leads
- Influence of leads in blockaded region?

1.2 Pauli spin blockade in double quantum dots

- Spin blockade - blocking of the charge current through the system due to arrangement of spin-dependent levels in the dot. [4, 5, 3, 6] Double dots in spin blockade regime can be used to experimentally measure spin relaxation time (Johnson'05 [7])



(a) Consequent transport Right → Left through a double dot. Initial state has one electron (with arbitrary spin) on the Left island.
Single electron steps:

- R Lead → R Dot
- R Dot → L Dot (can be blocked)
- L Dot → L Lead (recovery of initial state)

(b) Energy level representation of transport through the dot.
Intermediate Singlet: current flows $(1,0)\to(1,1)_S \rightsquigarrow (2,0)_S \to (1,0)$
Intermediate Triplet:
$(1,0)\to(1,1)_T\to\dots$ is blocked
$(1,1)_T \nrightarrow (2,0)_T$ (energy)
$(1,1)_T \nrightarrow (2,0)_S$ (spin)

- Once current is blocked - it will take spin-flip ((1,1) triplet-singlet relaxation) time to unblock it [8]

2 Model and equations

2.1 Spin relaxation due to leads

- We assume that the intrinsic relaxation times are slow and the main spin-flip process is due to electron exchange with the leads : spin up leaves the dot and spin down enters back

$$\mathcal{H} = H_{\rm dot} + H_{\rm leads} + V$$

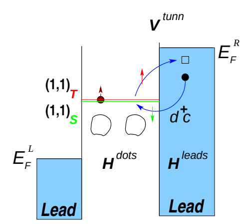
$$H_{\rm leads} = \sum_{\alpha=L,R}\sum_{\mathbf{k},\sigma}\xi_{\alpha\mathbf{k}}c^\dagger_{\alpha\mathbf{k}\sigma}c_{\alpha\mathbf{k}\sigma}$$

- free electrons

$H_{\rm dot}$ - localized energy levels in the dots

$$V = \sum_{\alpha=L,R}\sum_{\mathbf{k},\sigma}\left(W_{\alpha k}d^\dagger_{\alpha\sigma}c_{\alpha\mathbf{k}\sigma} + W^*_{\alpha k}c^\dagger_{\alpha\mathbf{k}\sigma}d_{\alpha\sigma}\right)$$

- tunneling Hamiltonian

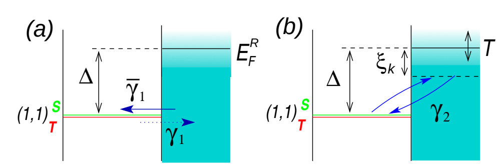


Rate equations from density matrix

- Density matrix equation $\dot{\rho}_i = -\sum_{f({\rm out})}\Gamma_{fi}\rho_{0i} + \sum_{f({\rm in})}\Gamma_{if}\rho_{0f}$
- Transition rates between electron states (leads × dot) : $|\,i\,\rangle = |\,e_i\,\rangle\times|\,{\rm dot}_i\,\rangle$

$$\Gamma_{fi} = 2\pi\delta(\epsilon_i-\epsilon_f)\left|V_{fi}+\sum_m\frac{V_{fm}V_{mi}}{\epsilon_i-\epsilon_m}+\dots\right|^2$$

- Environment (leads) relax much faster than dot states: define transition rates between dot states: $\boxed{\gamma_{fi} = {\rm Tr}_{e_i,e_f}\,\Gamma^{fi}\rho^0_{e_f}\rho^0_{e_i}}$



(a) First order: one-step processes $(1,1)_{S,T}\leftrightarrow(1,0)$

$$\boxed{\bar{\gamma}_1 = \Gamma_R f(\Delta)\,,\quad \gamma_1 = \Gamma_R[1-f(\Delta)]}$$

(b) Second order: two jumps $(1,1)_{S,T}\leftrightarrow(1,1)_{S,T}$

$$\boxed{\gamma_2 = \frac{2\pi}{\hbar}\sum_{\mathbf{k}}\mathcal{N}_F|W_{R,k}|^4\left|\frac{1}{\Delta-\xi_k+i0}\right|^2 f(\xi_k)[1-f(\xi_k)] \approx \Gamma_R\frac{T\,T_R}{\Delta^2}}$$

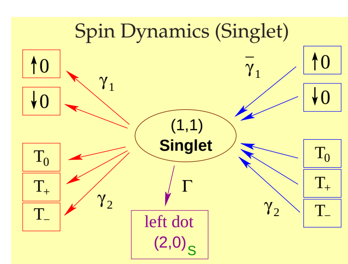
Here $T_R = \dfrac{\hbar\Gamma_R}{2\pi}$, $\Gamma_R = \dfrac{2\pi}{\hbar}\mathcal{N}_f\,|W_{R,k}|^2$, and $f(\xi_k)$ - Fermi function.
Similar to co-tunneling, [9, 10]

2.2 Rate equations

- Minimal number of states:

$\lvert\uparrow 0\rangle$	$\uparrow_L$	$P_{\uparrow 0}$
$\lvert\downarrow 0\rangle$	$\downarrow_L$	$P_{\downarrow 0}$
$\lvert S\rangle$	$\frac{1}{\sqrt{2}}(\uparrow_R\downarrow_L - \downarrow_R\uparrow_L)$	P_S
$\lvert T_0\rangle$	$\frac{1}{\sqrt{2}}(\uparrow_R\downarrow_L + \downarrow_R\uparrow_L)$	P_{T_0}
$\lvert T_+\rangle$	$\uparrow_R\uparrow_L$	P_{T_+}
$\lvert T_-\rangle$	$\downarrow_R\downarrow_L$	P_{T_-}

P_i - probability to find the dot in state i



$P_{\uparrow 0}+P_{\downarrow 0}+P_S+P_{T_0}+P_{T_+}+P_{T_-}=1$ - Normalization

$$\dot{P}_S = -(\gamma_1+\frac{3}{2}\gamma_2)P_S + \frac{1}{2}\bar{\gamma}_1(P_{\uparrow 0}+P_{\downarrow 0}) + \frac{1}{2}\gamma_2(P_{T_0}+P_{T_+}+P_{T_-}) - \Gamma P_S\,,$$

$$\dot{P}_{T_0} = -(\gamma_1+\frac{3}{2}\gamma_2)P_{T_0} + \frac{1}{2}\bar{\gamma}_1(P_{\uparrow 0}+P_{\downarrow 0}) + \frac{1}{2}\gamma_2(P_S+P_{T_+}+P_{T_-})\,,$$

$$\dot{P}_{T_+} = -(\gamma_1+\gamma_2)P_{T_+} + \bar{\gamma}_1 P_{\uparrow 0} + \frac{1}{2}\gamma_2(P_S+P_{T_0})\,,$$

$$\dot{P}_{T_-} = -(\gamma_1+\gamma_2)P_{T_-} + \bar{\gamma}_1 P_{\downarrow 0} + \frac{1}{2}\gamma_2(P_S+P_{T_0})\,.$$

3 Results

3.1 Eigenmodes - spin/charge relaxaton

- Consider Left-Right dot in contact with Right lead and $\Gamma=0$ (no escape to Left dot);
  No magnetic field: introduce $P_0 = P_{\uparrow 0} = P_{\downarrow 0}$ and $P_{T_1} = P_{T_+} = P_{T_-}$
  Remove probability P_0 (unoccupied right dot) and diagonalize equations
- Eigenmodes of double dot system dynamics:

$$\dot{P}_\eta(t) + \Gamma_\eta P_\eta(t) = J_\eta\,,\quad \eta = 1,2,3:$$

$$\begin{array}{lll} P_1 = P_{T_0} - P_{T_1}, & \Gamma_1 = \Gamma_s,\ \ J_1 = 0 & \text{(spin);}\\ P_2 = 3P_S - (P_{T_0}+2P_{T_1}), & \Gamma_2 = \Gamma_s,\ \ J_2 = 0 & \text{(spin);}\\ P_3 = P_S + (P_{T_0}+2P_{T_1}), & \Gamma_3 = \Gamma_c,\ \ J_3 = 2\bar{\gamma}_1 & \text{(charge).}\end{array}$$

$\Gamma_s = \gamma_1 + 2\gamma_2$ - spin relaxation

$\Gamma_c = \Gamma_R[1+f(\Delta)]$ - charge relaxation rate

- Note: electron exchange between single dot and a lead is described by $P_2 = P_\uparrow - P_\downarrow$ (with rate Γ_s) and $P_3 = P_\uparrow + P_\downarrow$ (with rate Γ_c).

3.2 Spin blockade current

- Current : escape $(1,1)_S \rightsquigarrow (2,0)_S$

$$\boxed{I(t) = e\Gamma P_S(t)} \Rightarrow \text{Stationary:}\ \boxed{I = e\,\frac{f(\Delta)}{1+f(\Delta)}\,\frac{2\,\Gamma_s\Gamma}{4\,\Gamma_s + \Gamma(3+\Gamma_s/\Gamma_c)}}$$

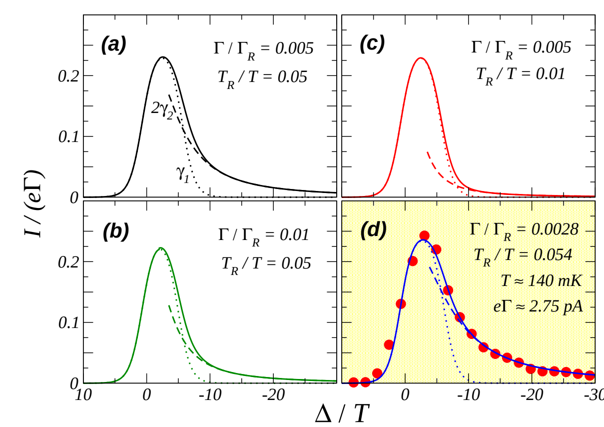
This expression describes the entire profile of the current in spin blockade, both peak (resonant exchange with the lead) and blockade valley. In these two limits I-expression also simplifies to

$$\Gamma_s \gg \Gamma:\qquad I = \frac{1}{2}e\Gamma\frac{f(\Delta)}{1+f(\Delta)},\quad \text{(peak)}$$

$$\Gamma_s \ll \Gamma:\qquad I = \frac{1}{3}e\Gamma_s,\quad \text{(valley, } f(\Delta)\approx 1)$$

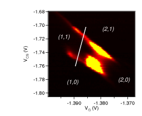
The valley dominated by algebraic tail: $\Gamma_s = \Gamma_R[1-f(\Delta)] + 2\gamma_2(\Delta)$ where $\gamma_2 = \Gamma_R^2\dfrac{T}{\Delta^2}\dfrac{\hbar}{2\pi}$

3.3 Current profiles : examples

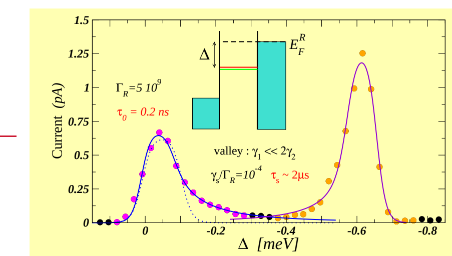


Current through a double dot system in the spin blockaded regime. The dotted (dashed) lines show asymptotes due to first (second) order processes that dominate peak(valley). Panel (d) shows a fit to measured current (circles) along a line-cut of the spin blockade peak reported in [3].

3.4 Spin relaxation times from experiment



(thanks to C.B.Simmons *et al.* [3])



- Peaks - direct exchange with leads, γ_1
- Tail - suppressed current, $2\gamma_2 \gg \gamma_1$
- Second peak - *hole* transport cycle:
  $(2,1)\to(1,1)_{(T\to)S}\to(2,0)_S\to(2,1)$
  with relaxation between $(1,1)_{T,S}$ states occuring via left lead and intermediate state $(2,1)$
- Extracted values :
  $T\approx 130\ mK$
  $\Gamma_0 \approx 10^9\ 1/s \to \tau_0 \sim 0.2\ ns$
- Blockade :
  spin-flip time
  $\tau_s \sim 2\ \mu s$

Conclusions

- We presented a simple theory for spin relaxation via leads
- It explains the current profile in spin-blockade experiment on Si : peaks and valley
- Electron exchange with the leads may dominate *intrinsic* mechanisms (HF and SO) of spin relaxation in dots
- Triplet-Singlet relaxation rate (for a given experimental geometry/coupling to the leads Γ_R):
  peaks $\quad\tau_{s0}\sim 0.2\ ns$
  blockade $\quad\tau_s \sim 2\ \mu s$.

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